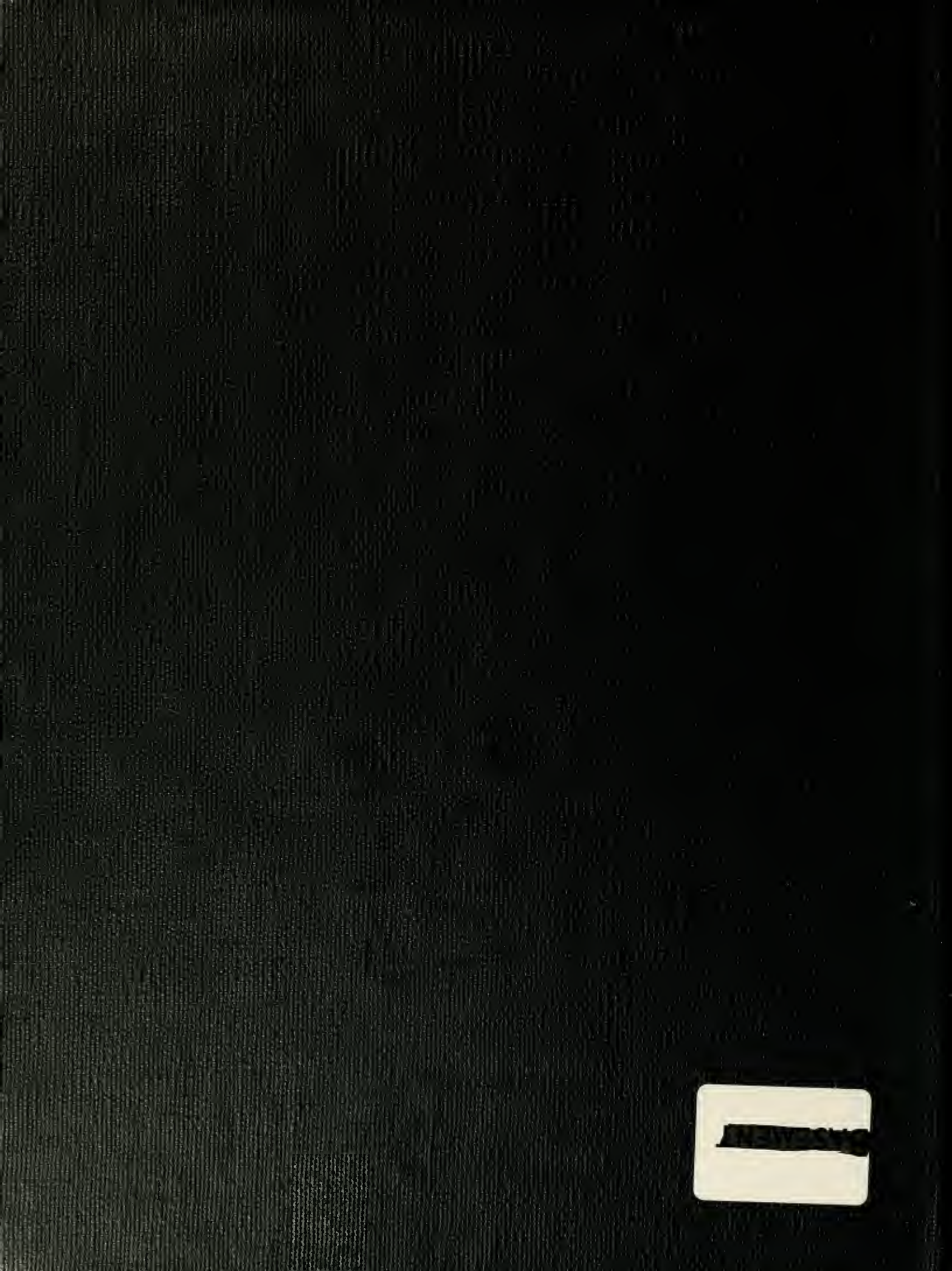

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SHADOW PRICES IN PROJECT EVALUATION, WITH AND WITHOUT DISTORTIONS, AND WITH MANY GOODS AND FACTORS*

Jagdish N. Bhagwati and Henry Wan, Jr.

Number 205 July 1977

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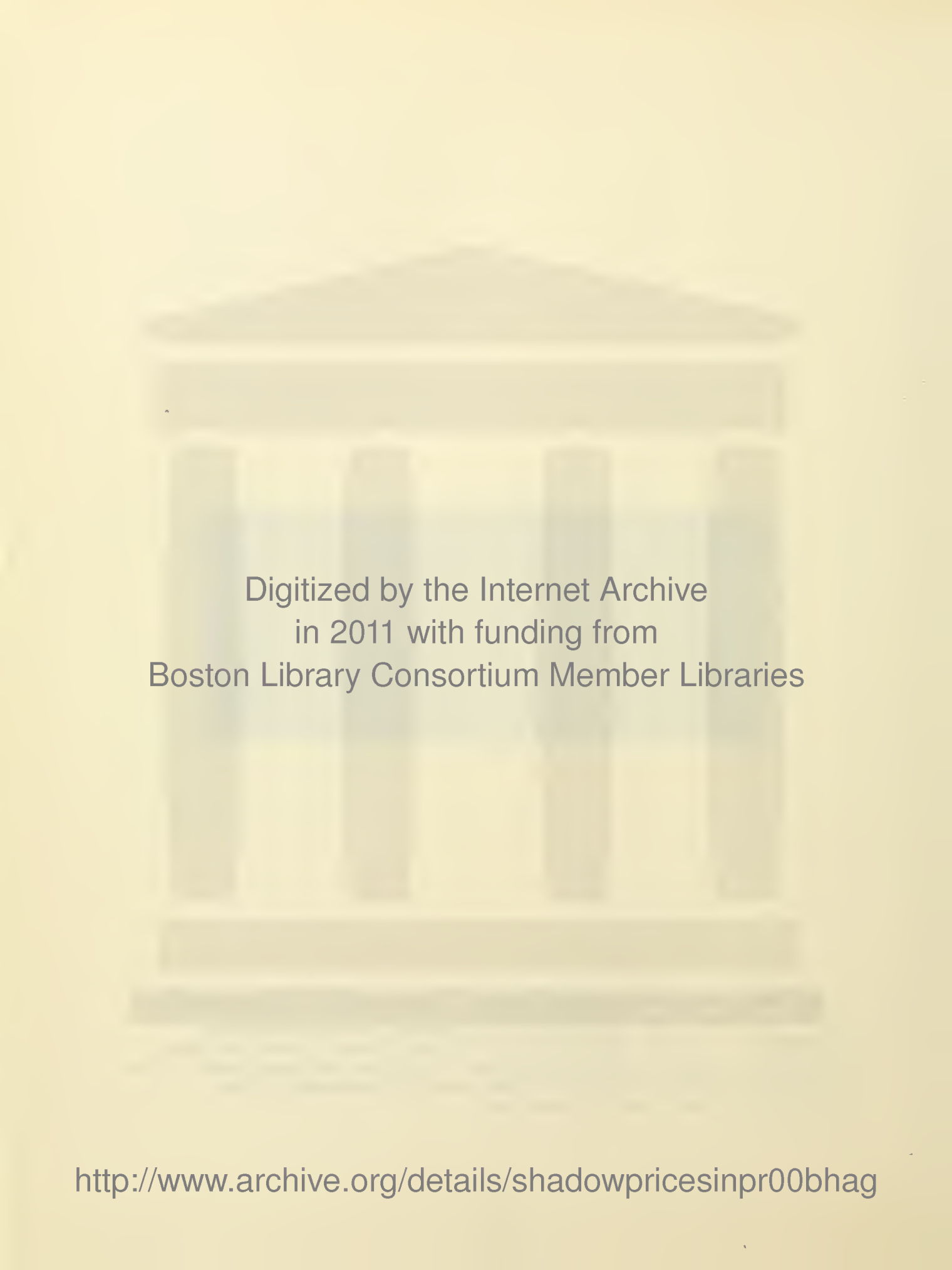
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The analytical literature on the derivation of shadow prices for use in project evaluation is a vast and ever-growing one. From the viewpoint of the focus of our present paper, to be stated below, it is conveniently classified according to the type of technology that the analyst assumes, this in turn reflecting the tradition within which the analyst works. Four such traditions may be distinguished:

1. The public-monopoly tradition: The sizeable literature in this tradition has been reviewed by Lesourne (1975). This literature is characterized by the assumption of strictly diminishing returns to scale everywhere, so that the derived shadow prices for project evaluation are necessarily applicable only to projects of negligible size (cf. Lesourne, 1975, Chapter 3).

2. The linear-programming/activity-analysis tradition: The representative literature here can be found in the celebrated work of Dorfman-Samuelson-Solow (1957). This literature is characterized by the assumption of fixed coefficients.

3. The public finance tradition: This tradition partly overlaps with the public monopoly tradition but has also developed recently in the works of public finance theorists such as Little, Mirrlees and Diamond, who predominantly use the constant returns to scale assumption with generalized substitution in production. The most celebrated, practical work in this area is the Little-Mirrlees (1972) Manual, written for the OECD and now widely used for project evaluation; and a recent, excellent example of theoretical work in this area is the paper of Diamond and Mirrlees (1976).

4. The trade-theoretic tradition: This tradition is of relatively

recent vintage and, starting with relatively early contributions by Joshi (1972), Lal (1974) and Corden (1974)--all of whom were trying to develop the theoretical rationale behind the Manual rules--it has received major prominence with the recent, important and elegant work of Findlay and Wellisz (1976) and later by Srinivasan and Bhagwati (1976). This work of Findlay-Wellisz-Srinivasan-Bhagwati (FWSB) explicitly deploys the tools, insights and ideas of general-equilibrium international trade theory. This work is therefore characterized by constant returns to scale and generalized substitution in production: technological features of the traditional models of trade theory.

In particular, the Findlay-Wellisz-Srinivasan-Bhagwati (FWSB) analyses have been addressed to the question of deriving the shadow prices for primary factors, for the purpose of project evaluation, in the presence of distortions: Findlay-Wellisz considering product-market and trade distortions and Srinivasan-Bhagwati also extending their analysis to a number of factor market distortions.

Their analyses having been conducted essentially within the framework

of the 2x2 model of traditional international trade theory, an important consequence is what might be called the "stationariness" of the shadow prices of factors such that, as Srinivasan-Bhagwati phrased it, "while we have confined our analysis to "small" projects, drawing infinitesimal resources away from the existing distorted situation, it is equally clear from our analysis that the results will also hold for "large" projects. Given the Rybczynski-line properties of the different models, the shadow prices of factors will be identical for small and large shifts of factors into the project " (1976, page 23).

When the Rybczynski-line properties no longer hold, the "marginal variational" shadow prices applicable for single projects with infinitesimal factor withdrawals will vary as the factor endowment vector varies. Similarly, for a project withdrawing finite amounts of factors[1] the "marginal variational" shadow prices computed before the withdrawal will differ from those computed after the withdrawal; therefore, project evaluation with the former prices will understate the opportunity cost while an evaluation using the latter will overstate it, as a consequence of the Law of Variable Proportions. Moreover, shadow prices computed by marginal variations from the "residual factor vector" (after the withdrawal) will depend upon the size and composition of the factors withdrawn. [As a matter of pure formality, "true" shadow prices can be defined, a posteori, for projects withdrawing finite factor dosages;[2] such shadow prices, used for project evaluation, will tautologically yield the opportunity cost. However, these prices will vary from project to project and their derivation will require each time the solution of a full programming problem for project

---

[1] Equally, for a successive sequence of "small" projects, collectively withdrawing finite amounts of factors in the aggregate.

[2] See Appendix I.

selection. Such shadow price computation will therefore become a purely academic exercise: the projects having been selected in the programming problem already![1]]

The "stationarity" of the shadow prices, in the presence of non-infinitesimal factor withdrawals, such that (quite generally) the valuation of these factors in project use at the marginal-variational shadow prices equals their social opportunity cost in terms of output foregone, is therefore a critical question, rather than a matter for idle and strictly-academic speculation.[2] We clearly need to examine the circumstances in which stationarity will obtain, so that the derivation of shadow prices in project analysis is not confined to negligible projects and hence to negligible claim on our attention.

The present paper is therefore addressed to examining the outcome regarding the stationariness of shadow factor prices once the assumptions underlying the Findlay-Wellisz-Srinivasan-Bhagwati (FWSB) analyses are relaxed and to analyzing the degree of bias introduced by using such stationary shadow prices. Section I basically recapitulates the substance of the FWSB analysis, essentially retaining the 2x2 model and the Samuelson "small-country" assumption of fixed international prices, but distinguishing

---

[1] A similar point has been made by Bhagwati and Srinivasan (1973) in relation to estimating the effective-rate-of-protection (ERP) index for resource-allocation prediction in the case of generalized factor substitution. They argue that, in general, to compute the "correct" ERP index, we must solve the general-equilibrium system for the tariff change; but if we have done that, we already know the total resource-allocation change and we do not need the ERP index to tell us the direction of such change.

[2] For an extremely scathing and articulate critique of cost-benefit analysts by a programmer-planner who argues that shadow prices which apply to negligible (i.e. infinitesimal) projects are of negligible interest, see Rudra (1972). He is clearly assuming what we christen here the "non-stationarity" of the marginal-variational shadow factor prices and therefore his critique, while fundamentally sound in principle, goes too far in failing to show awareness of possible stationarity of shadow prices.

between the case without a (trade or product-market) distortion and that with it.[1] Section II retains the 2x2 model and the small-country assumption but briefly considers an alternative trade distortion (a quota) that, unlike the FWSB trade distortion, destroys the stationarity of shadow factor prices. Section III then reverts to the cases without a distortion or with only the ("nice") FWSB distortion, retains the small-country assumption, and examines the effect of going beyond the 2x2 model. Our analysis, kept deliberately simple here and omitting mathematical argumentation (whish is relegated to Appendix I), extends to different possible cases: Case I: Goods (m) > Factors (n), no distortion; Case II: Goods (m) = Factors (n), no distortion; Case III: Goods (m) < Factors (n), no distortion; Case IV: Goods (m) > Factors (n), with (FWSB) distortion; Case V: Goods (m) = Factors (n), with (FWSB) distortion; and Case VI: Goods (m) < Factors (n), with (FWSB) distortion. Section IV offers concluding observations, indicating the applicability of our analysis to other problems in trade theory (e.g., the transfer problem and the welfare effects of labor mobility) and the relationship of our results to mathematical programming. Appendix I contains the formal proofs of the major propositions established in Section III whereas Appendix II brings together some well-known results concerning the effects of consumption distortions on welfare evaluation and offers some pertinent observations on the effects of relaxing the small-country assumption that underlies our, and in fact earlier, analyses.

---

[1] While FWSB focus only on the distortion case, our present analysis, going beyond the 2x2 model, makes even the nondistortion case worthy of attention, as will be evident below.

Section I: Recapitulating and Completing Analysis within the FWSB Model

The FWSB model is characterized by three key features: constant-returns-to-scale production functions;[1] two primary factors producing two traded goods (the so-called 2x2 model of trade theory); and fixed foreign prices for the two traded goods (the so-called Samuelson small-country assumption). The problem of deriving shadow factor prices for a project producing a third traded good then is tantamount to deriving the changes in outputs of the two traded goods (x_1 and x_2) that follow from the withdrawal of factors (v_1 and v_2) from existing allocations and then evaluating these output changes at (the fixed) international prices (which, of course, represent the true opportunity costs).

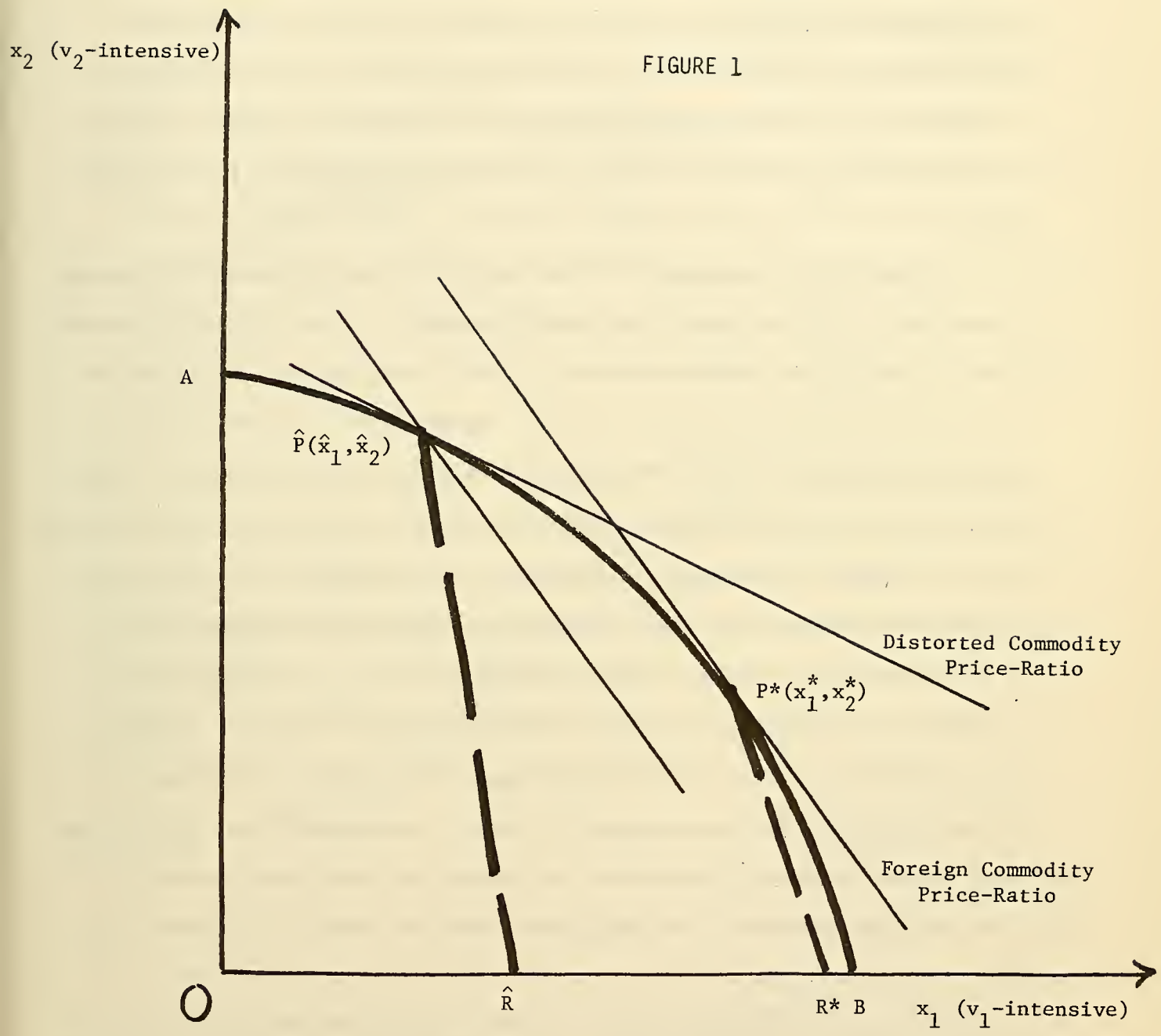
Thus, take Figure 1 which depicts the usual production possibility curve for the two traded goods, x_1 and x_2. Distinguish now two cases: (1) no distortion; and (2) a (given) product market distortion.[2] For the former case, production will take place at P*; for the latter case, it will occur at $\hat{P}$. In the former case, the withdrawal of factor v_2, assumed to be intensively used in commodity x_2, will successively reduce the output of x_2 and increase it for x_1 along the Rybczynski line P*R* whereas a similar withdrawal of factor v_2 will define, in the latter case, a similar

---

[1] FW and SB do not explicitly rule out factor-intensity reversals. On what happens when they are present, see our analysis below.

[2] The product market distortion essentially implies that the domestic producers face production tax-cum-subsidy inclusive prices or tariff-inclusive prices, rather than the international prices for the two goods. If we invoke a tariff, there will be a consumption distortion as well. But this creates no difficulties for our analysis, so that our analysis holds equally for trade distortions; see, however, Appendix II for further discussion of this problem.

FIGURE 1


x_2 (v_2-intensive)
A
$\hat{P}(\hat{x}_1, \hat{x}_2)$
Distorted Commodity Price-Ratio
$P^*(x_1^*, x_2^*)$
Foreign Commodity Price-Ratio
O
$\hat{R}$
R* B
x_1 (v_1-intensive)

(non-parallel) Rybczynski line $\hat{P}\hat{R}$.[1] In each case, the commodity price-ratio is clearly held constant as the factor is withdrawn for use in the project producing commodity x_3: in the no-distortion case this is the international price-ratio whereas, in the distortion case, it is the distorted commodity price-ratio. The evaluation of the changes in outputs along the relevant Rybczynski line, however, is at the international prices in both cases.

An important consequence is that, as long as the withdrawal of factors, no matter how large, permits the "small" economy to remain on the Rybczynski line in Figure 1, whether there is a given product market distortion or none, the shadow factor prices will be "stationary."[2] What happens if factors are withdrawn such that the economy moves off the Rybczynski line? Focusing on the no-distortion case (with everything applicable, qualitatively, to the with-distortion case), assume that the withdrawal of primary factor v_2 leads the economy finally to complete specialization on commodity x_1 at the bottom of P*R* at R*. If then another unit of v_2 is withdrawn, it is evident that the economy will move towards the origin along the horizontal axis and that, as it does so, the real rental of factor v_2 will rise in terms of both of the traded goods, x_1 and x_2. Essentially, one has slipped out of the "linearity" property of the system and diminishing returns to varying proportions are now taking over. The net result therefore is that,

---

[1] The slope of the Rybczynski lines reflects the average productivity of the primary factors whereas the slope of the commodity price-ratio at which the Rybczynski line is defined reflects, of course, the marginal productivity of the factors. On this, as also on the relationship between the Rybczynski lines at different points on the production possibility curve (as at P* and $\hat{P}$), see Bertrand-Flatters (1971) and Brecher (1974), in particular.

[2] SB do not consider what happens beyond the Rybczynski line. They rather consider other questions such as the relationship of their results on shadow factor prices to the use of Domestic Resource Costs and Effective Rate of Production as project-evaluation criteria.

for any withdrawals of the primary factor v_2 beyond those that lead from P* to R*, the shadow prices of the factors are no longer stationary and can be shown to increase (for the factor withdrawn) for successive withdrawals of the factor for utilization in the project in question. This also implies that, for these ranges of increasing-cost withdrawal of factor v_2, the use of the shadow prices for marginal variation at P* would yield an <u>understatement</u> of the true "shadow" cost of the factor. Alternatively, one may phrase this to say that the use of "marginal-variational" shadow prices, when stationariness does not obtain, ignores the "secondary cost" that must be added to the "primary cost" as measured at the "marginal-variational" shadow prices.

All this can be seen perfectly generally, for withdrawals of <u>both</u> factors, in terms of the McKenzie-Chipman diversification cone in Figure 2, by confining ourselves for simplicity to the no-distortion case. Assuming that q_1 and q_2 define the v_1/v_2 ratios chosen at the prevailing commodity price-ratio (i.e. at P* in Figure 1) and the associated factor price-ratio q_1q_2, note that the overall factor endowment ratio must be a weighted sum of the two sectoral factor proportions.[1] It is evident then that q_1Oq_2 defines a diversification cone: as long as the factor endowment vector lies strictly <u>within</u> the cone, both goods will be produced at the postulated commodity and associated factor price-ratios. Hence the aggregate endowment vector $(\bar{v}_1,\bar{v}_2)$ is shown to lie within the cone q_1Oq_2, indicating production of both goods at levels x_1^* and x_2^* (correspondong to P* in Figure 1). Note

---

[1] Thus, denoting the overall endowments as $\bar{v}_1$ and $\bar{v}_2$, we must have (assuming full employment) $\bar{v}_1/\bar{v}_2$ as the v_2-share-weighted sum of the v_1/v_2 ratios in the production of the two goods.

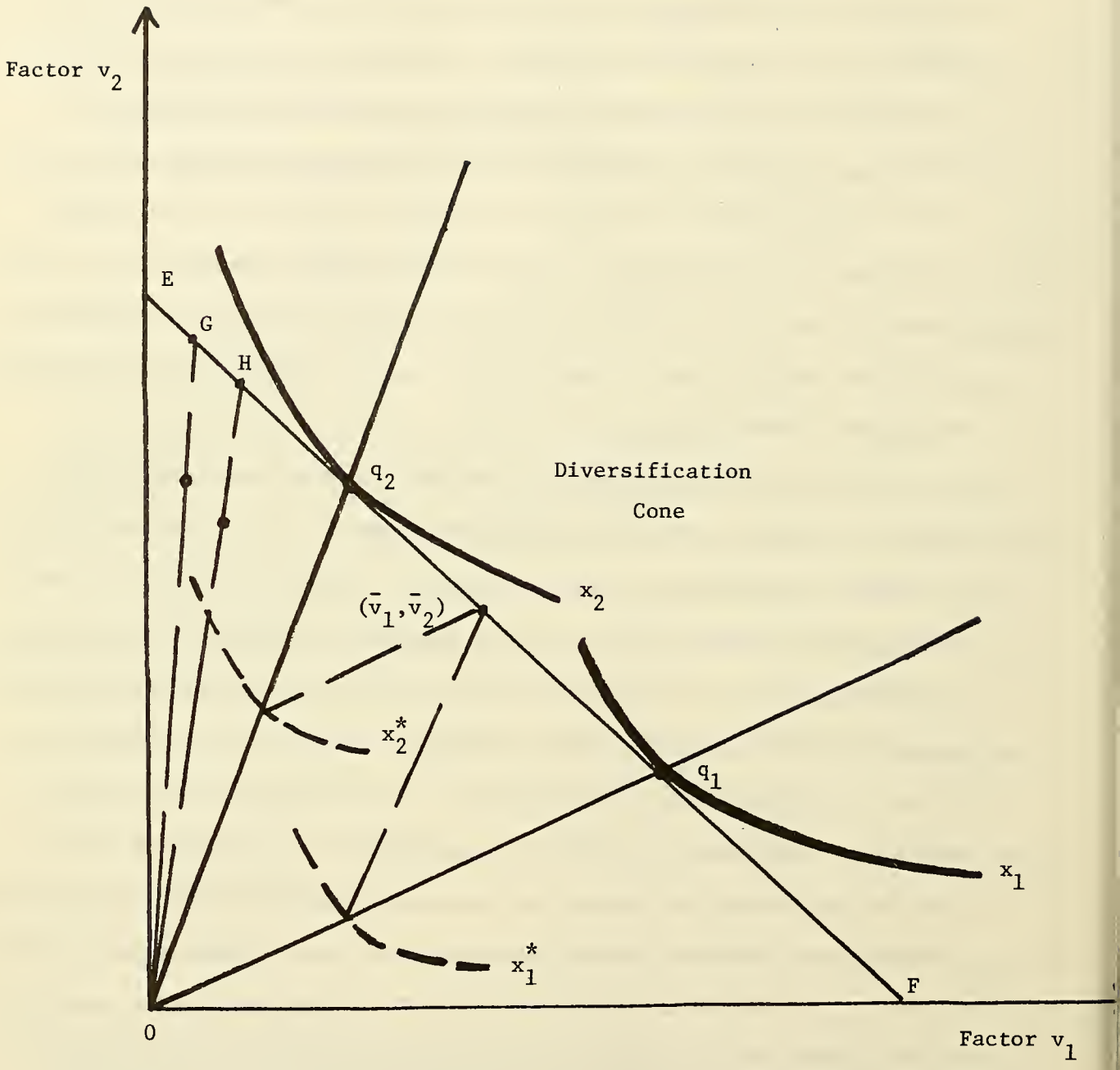

Factor v_2
E
G
H
q_2
Diversification
Cone
x_2
$(\bar{v}_1, \bar{v}_2)$
x_2^*
q_1
x_1
x_1^*
F
0
Factor v_1

Figure 2

that the aggregate endowment vector $(\bar{v}_1, \bar{v}_2)$ is shown, by the parallelogram in Figure 2, to be a weighted vector sum of the factor proportions in the two goods, the actual outputs being read off from the isoquants x_1^* and x_2^*.

It follows immediately that (given the commodity price ratio), as long as the residual factor vector, left over after successive withdrawals of the factors for project-use, continues to lie within the cone $q_1 0 q_2$, the equilibrium factor price-ratio need not change from $q_1 q_2$ and all changes in factor endowments will be accommodated merely by changes in the composition of outputs. Hence, the McKenzie-Chipman diversification cone, in the present case, is identical to what we shall refer to as the Rybczynski cone: the latter being the set of residual factor vectors for which shadow factor prices will remain stationary at $q_1 q_2$. This stationarity of factor prices will disappear, however, as soon as the residual factor vector slips out of this cone.[1] Note also the following:

(i) As must be evident, $q_1 q_2$ represents the "marginal-variational" shadow factor prices, these being the prices corresponding to infinitesimal factor withdrawal.

(ii) The secondary cost of withdrawn factors that leave the residual factor vector outside the Rybczynski cone, defined as the excess of their true shadow cost over their cost if measured at the marginal-variational prices, will be an increasing function of the extent to which the residual factor vector slips out of the Rybczynski cone. For the two-factor case of Figure 2, this is readily illustrated in Figure 3 as follows. Thus, in Figure 2, first project every residual factor vector into the common tangent line

---

[1] Reverting to Figure 1, note then that all withdrawals of factor v_1, resulting in moves along P*R*, leave the residual factor vector in the diversification cone whereas further withdrawals of v_1, leading to moves along R*0, imply that the residual factor vector has slipped out of the cone.

EF: two such projections being at G and H. Take ratios $\frac{EE}{EF},\ldots\frac{EG}{EF},\ldots$ $\frac{EH}{EF},\ldots\frac{Eq_2}{EF},\ldots\frac{Eq_1}{EF},\ldots\frac{EF}{EF}$: these range from 0 to 1. Plot these along the horizontal axis of Figure 3. The vertical axis then plots the corresponding values of the ratio (t) of the output value producible by a residual factor vector to its marginal-variational valuation. Thus, for projects which leave residual factor vectors with projection at G in Figure 2, the t-value is correspondingly at G' in Figure 3 and, for H (which is closer to the Rybczynski cone), the t-value is higher at H' in Figure 3, indicating a reduced secondary cost. For the projections lying between q_1 and q_2, the t-value is 1: there is no secondary cost. For projections beyond q_1, up until F (where the project withdraws all of factor v_2 and leaves only factor v_1 behind), the t-value will again fall successively: thus showing afresh that the secondary cost rises as the residual factor endowment gets further away from the Rybczynski cone. The flat-top curve of t-value that results in Figure 3 may then be described as the Guyot curve, Guyot being the term used by the oceanographer Hess (1946) for flat-topped seamounts.[1]

(iii) Finally, note that the unique Rybczynski cone in Figure 2 implicitly rules out factor-intensity reversals. If we had allowed for such reversals, however, we would have had more than one diversification cone. Thus, with two diversification cones, for example, the guyot profile in Figure 3 would have had another linear (and upward-sloping) segment--as illustrated, with broken lines, for the case where such a segment would lie somewhere between points 0 and q_2; and nothing substantive in our analysis would need to be changed.

---

[1] The guyot profile can be more generally derived by plotting (instead) on the horizontal axis the cost share of one factor in total national income: this would permit us to consider as many factors as necessary. This technique has been set out in an unpublished mimeographed paper and is available from us on request.

Guyot Profile

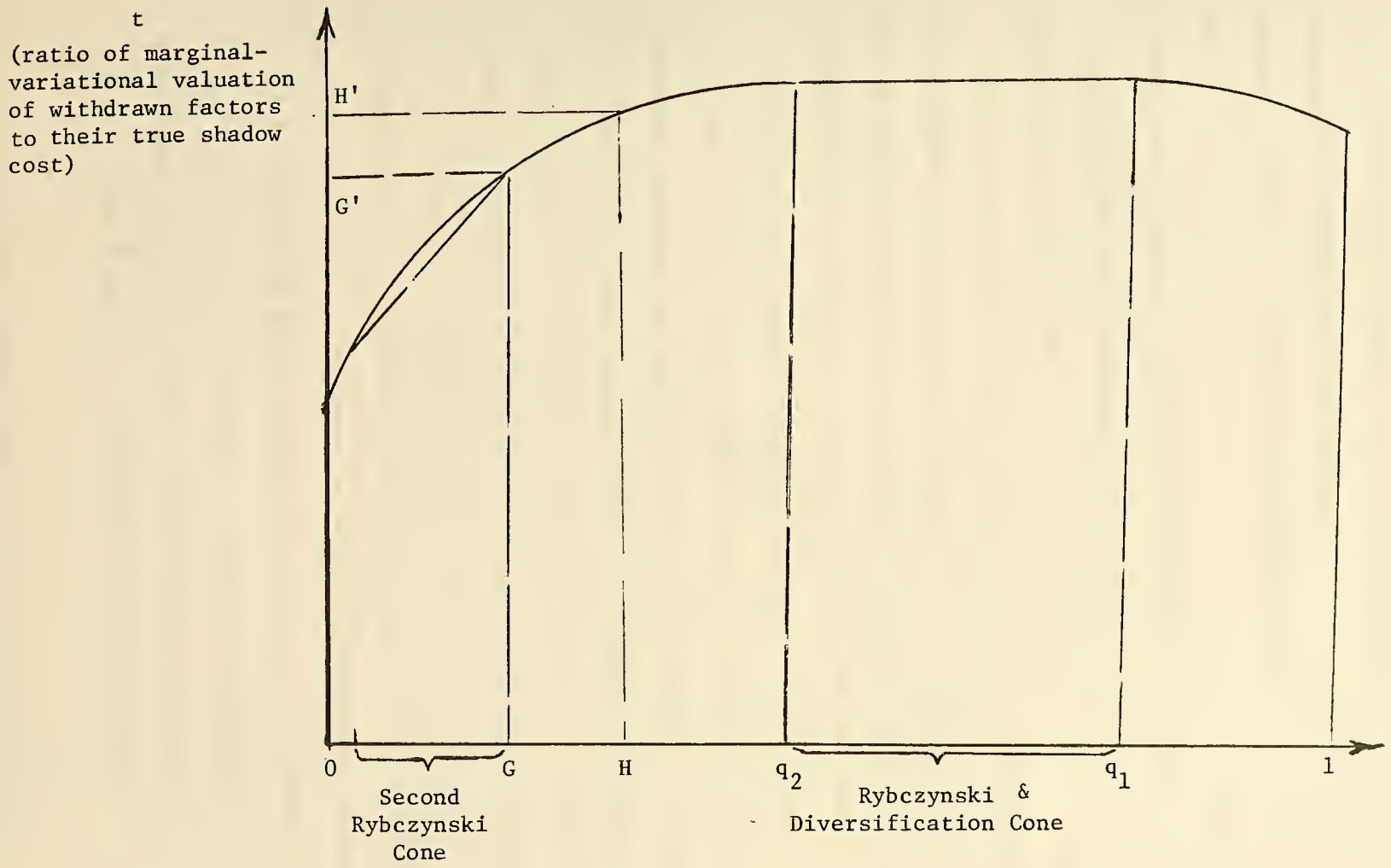


Figure 3

Section II: A Case of Variable Distortion: Nonstationary Shadow Prices

A digression into nonstationariness of shadow factor prices, arising in the with-distortion case from the nature of the assumed distortion, is now in order, to highlight the critical nature of the FWSB product-market distortion.

Thus, consider the case where the distortion arises from a quantitative restriction rather than, as in the FWSB analysis, from an ad valorem restriction. Thus, assume that, instead of a tariff (with consumption distortion therefrom suitably offset), one has an import quota. In Figure 4, if the initial, distorted equilibrium is at $\hat{P},\hat{C}$, then the FWSB analysis implies that the corresponding distorted commodity price-ratio will remain unchanged as factor v_2 is withdrawn and the economy therefore slides down the Rybczynski-line $\hat{P}\hat{R}$. Thus, if the residual factors lead the economy to $\hat{Q}$, then C would be the corresponding consumption point.[1] However, if the tariff were to be replaced by a quota, such that imports of commodity x_2 cannot exceed $\hat{C}\hat{S}$, then the new equilibrium will rather be at $\hat{\hat{Q}},\hat{\hat{C}}$, with imports at $\hat{\hat{C}}\hat{\hat{S}} = \hat{C}\hat{S}$. The points $\hat{Q}$ and $\hat{\hat{Q}}$ will lie on the production possibility curve for the residual factor endowment, of course; the implied "tariff-equivalent" distortion in this new equilibrium will have increased (being measured as the discrepancy between the new, distorted commodity price-ratio and the fixed foreign price-ratio); and $\hat{\hat{Q}}$ will lie on the Rybczynski-line $\hat{\hat{P}}\hat{\hat{R}}$ defined at the new, distorted commodity price-ratio. Note that the total move from $\hat{P}$ to $\hat{\hat{Q}}$ can be decomposed in two different ways: either as a move along AB from $\hat{P}$ to $\hat{\hat{P}}$ for the distortion change and then from $\hat{\hat{P}}$ to $\hat{\hat{Q}}$ for the

---

[1] All C's are tangencies between social indifference curves and the foreign price-line (since consumption occurs at foreign prices, the tariff being offset by a consumption tax-cum-subsidy for this purpose, as noted above).

factor withdrawal; or as a move down to $\hat{Q}$ for the factor withdrawal and then from $\hat{Q}$ to $\hat{\hat{Q}}$ for the distortion change.

It is evident then that the quantitative restriction implies a variable distortion and hence, in general, the stationariness of the shadow factor prices will disappear (depending now, not just on the linear Rybczynski-line but also on shifts along the production possibility curve).[1] Quantitative trade restrictions are, of course, important in many developing countries and do therefore create the difficulty, just noted, for project analysis with shadow prices.

We might note that, equally, nonstationariness of shadow factor prices will generally arise from the relaxation of the small-country assumption, since varying foreign prices will have effects similar to those diagnosed in Figure 4.[2]

---

[1] Note that, in the above analysis of the quota-restriction, the effect of the quota on consumption distortion was being eliminated by assuming that the consumption would be allowed to take place at international prices. However, if consumption also occurs at the distorted prices, then the variable distortion implied by the quota also means that the consumption cost from the distortion can (and, in the illustration in Figure 4, will) increase so that one cannot any longer infer, a la Little-Mirrlees and the criterion followed in this paper and in the FWSB analysis, that the project should be accepted if it increases the total value of output evaluated at international prices: the analyst cannot now assume that there is a monotonic relationship between welfare and the distance of the availability locus (at international prices) from the origin.

[2] For further analysis of the large-country case, see Appendix II.

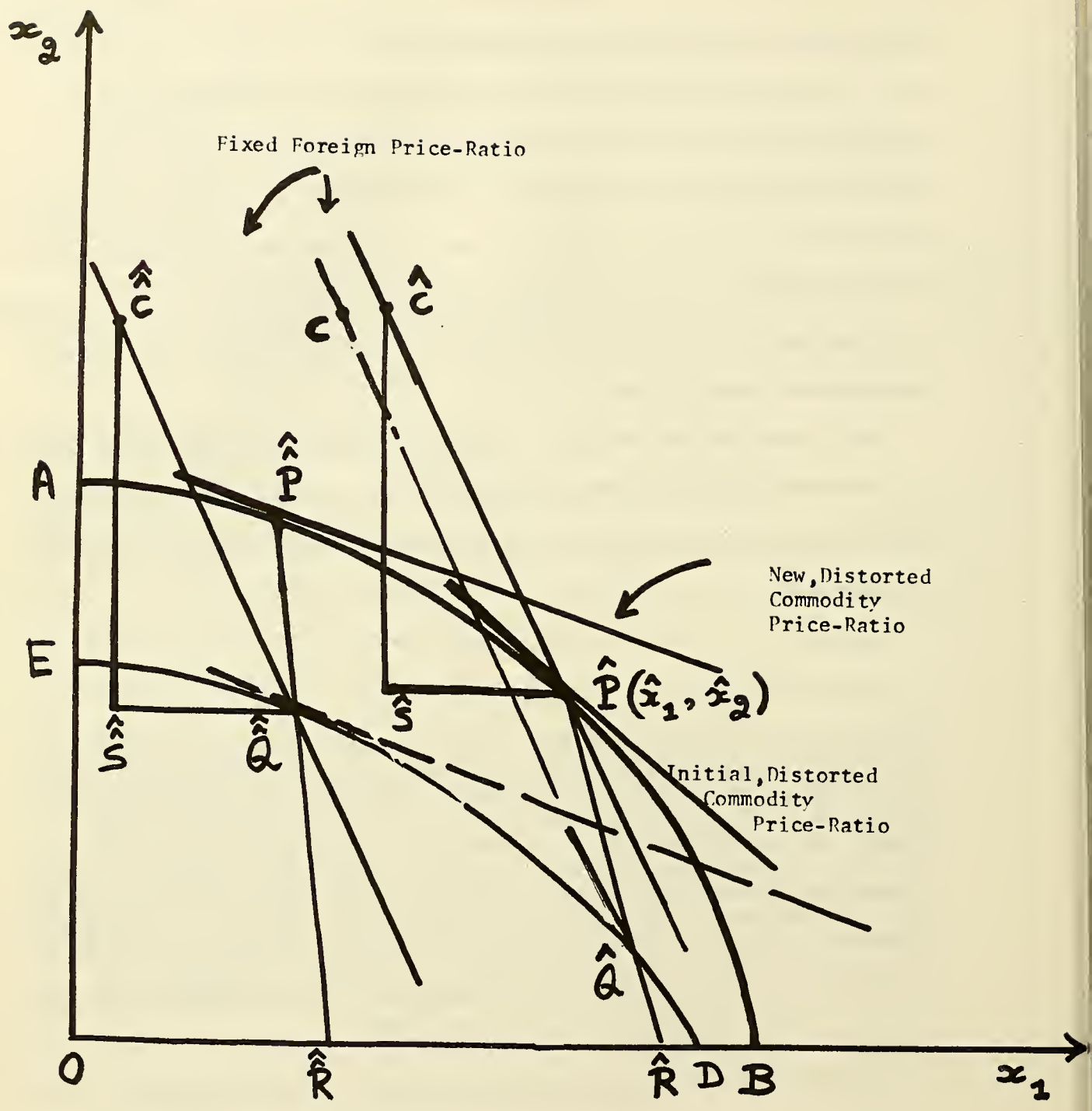

x_2
Fixed Foreign Price-Ratio
$\hat{\hat{C}}$
$\hat{C}$
C
$\hat{C}$
A
$\hat{\hat{P}}$
New, Distorted
Commodity
Price-Ratio
E
$\hat{P}(\hat{x}_1, \hat{x}_2)$
$\hat{\hat{S}}$
$\hat{\hat{Q}}$
$\hat{S}$
Initial, Distorted
Commodity
Price-Ratio
$\hat{Q}$
O
$\hat{\hat{R}}$
$\hat{R}$
D
B
x_1

FIGURE 4

Section III: Many Goods and Factors, with and without FWSB Distortions

We may now examine the effect of introducing many goods and factors, while confining our attention to the cases without, and with only FWSB, distortions. For obvious reasons, algebraic formulation of our analysis is now desirable. It is possible however to develop our main results, at least at a less general and qualitatively insightful level, much more simply. This, we do; and the algebraic formulation, necessary for general formulations, is therefore separated into Appendix I. It may be useful to note, in advance of the analysis, that it leads to the following principal conclusions in Table 1 in regard to the uniqueness and stationariness of shadow factor prices, for the six alternative cases that were set out in the introduction:

Table 1: Six Alternative Cases: Outcomes Regarding Shadow Factor Prices

CASES	NO DISTORTION	FWSB DISTORTION
Goods Outnumber Factors	I. No problem	IV. Shadow Prices may be undefined
Goods Equal Factors	II. No problem	V. No Problem
Factors Outnumber Goods	III. Shadow Prices may be non-stationary	VI. Shadow Prices may be nonstationary

Recalling that the shadow price of factors withdrawn for project-use is the sum of the resulting changes in outputs valued at international prices, we may now write this shadow cost as:

$$c_k = -p^* \Delta_k x$$

where c_k denotes the shadow factor cost of a project (program) k, p^* is

the vector of international prices and $\Delta_k x$ is the vector of changes in non-project production in the country as a result of the withdrawal of factors for project use. [Here, as in Appendix I, x, p and p* will represent m-dimensional vectors denoting output quantities, (domestic) output prices facing the firms and international prices, respectively. And v, w and w* will be n-dimensional vectors representing factor quantities and factor prices related to domestic and international prices, respectively.] Noting that

$$c_k \ (= -p^*\Delta_k x) \ = \ -w^*\Delta_k v$$

for pre-project marginal-variational shadow factor prices w* and infinitesimal factor withdrawal for project use, we can then pose two questions of principal importance in this section:

(i) can the social opportunity cost, c_k, be uniquely defined; and

(ii) will the shadow factor prices be stationary for finite factor withdrawals, in the perfectly general sense that $-w^*\Delta_k v$ will equal $(c_k =) -p^*\Delta_k x$, i.e. the valuation of the withdrawn factors at the pre-project marginal-variational shadow prices will equal the social opportunity cost of the foregone output?[1]

---

[1] In answering these questions, we will generally ignore the "extreme" cases which provide exceptions to our propositions and analysis, so that we can avoid frequent repetition of caveats regarding them. Such "extreme" cases are reviewed readily for the 2x2 model, where:
(a) Certain projects will cause difficulties. For instance, consider a project using up resources of magnitude and proportions such that the residual vector is no longer in the same diversification cone as the initial vector. Either complete specialization will occur, or the residual vector may now be in another diversification cone (as may happen when factor-intensity is reversible); and shadow prices will change.
(b) Certain projects almost surely will cause no difficulty. Thus, consider the case where the diverted resources, and hence, the residual resources, are proportional to the initial resources.

continued...

It is evident that Cases II and V, where goods equal factors, will create no difficulty: the shadow prices will be unique and stationary, in the same way as in the 2x2 FWSB analyses, with or without the FWSB distortion. Similarly, it is clear that in Case I, where goods outnumber factors and there is no distortion, the analysis of shadow prices is identical to that for the 2x2 model: a diversification cone can be defined readily and, within it, the shadow prices are unique and stationary.[1] However, problems arise in the three other cases, which we analyze successively.

(A) Case IV: Goods Outnumber Factors plus FWSB Distortion

We begin with Case IV where, while there is no problem as long as there is not distortion, the introduction of (even the "simple") FWSB distortion leads to difficulties, paradoxically. Intuitively, it is easy to see why.

For, the diversification cone is defined with respect to p, the domestic output prices. Hence, within the cone, the indeterminacy of the

---

(c) Certain circumstances almost surely will cause difficulties. For instance, take the case where the two unit-output isoquants, one for each product, are tangential to a common tangent at the same point.
(d) Certain circumstances almost surely will cause no difficulties. For instance, consider the case where, over some relevant range, the two inputs are "perfect substitutes" in the sense that a linear segment of the unit-output isoquant of at least one product prevails.

[1] The product-mix is indeterminate, of course, but this does not affect the shadow factor prices, as should be evident by redrawing the Lerner-Findlay-Grubert diagram, Figure 2, for more than two goods by merely putting in more isoquants tangent to the linear segment q_2q_1.

output mix is still compatible with unique and stationary factor valuations as long as the marginal-variational changes in outputs from factor withdrawals are evaluated at the domestic prices. However, the shadow factor prices require these output changes to be evaluated at the international goods prices, p^*. When $p = p^*$ (i.e., Case I where there is no FWSB distortion), therefore, shadow factor prices will also be unique and stationary within the diversification cone.

But when $p \neq p^*$, as with the FWSB distortion in Case IV, the shadow factor price for any factor withdrawal, even when the residual vector remains in the diversification cone, will not be unique but will reflect the particular product-mix happening to obtain out of the indeterminate many. Nor will the shadow factor prices, for that reason, be stationary for different factor withdrawals, all with residual vectors in the cone.

Suppose, however, that a "planner" chooses, for each residual vector, that particular output-mix whose value, evaluated at international prices p^*, is maximal. Could we then argue that, in this event, the shadow factor prices will be stationary within the diversification cone? The answer unfortunately is again in the negative, generally speaking. Note that, even if it were in the affirmative, to use such shadow prices for project evaluation, we would have to assume that these maximal-value product-mixes were in fact the equilibrium output-mixes obtaining in the economy before and after the factor withdrawals for the projects in question: otherwise, the "true" opportunity costs of the withdrawn factors would not correspond to the "shadow" opportunity costs as calculated with the maximal-value procedure.

All this should be perfectly intuitive, once the difference between p and p^* under the FWSB distortion is grasped. It is established more

formally in Appendix I but may also be illustrated to great advantage with the aid of the "micro-theoretic" diagrams, Figures 5(a) to (e). Figures 5(a)-(c) introduce the basic technique, concentrating on the no-distortion case, while Figures 5(d)-(e) illustrate then our basic propositions with this technique by using a 3-good, 2-factor FWSB-distortion depiction.

In Figure 5(a), two inputs v_1 and v_2 are used to produce output x, which is depicted along the vertical axis 0x. At the initial endowment v, height $v\bar{v}$ of the production surface OCD shows the output value before factor withdrawal for project use. A project using both factors in proportion to the initial endowments will leave a residual vector OB within the ray $0\bar{v}$. The difference in height between $B\bar{B}$ and $v\bar{v}$ reflects then the project cost. The unique tangent plane OPP' to surface OCD at $\bar{v}$ represents clearly the marginal-variational valuation of any residual factor vector.[1] Since $\bar{B}$ lies both on surface OCD and on plane OPP', such valuation incurs no secondary cost for the project in question. By contrast, a project using factors <u>not</u> proportional to the initial endowments will leave a residual vector 0A off the 0v ray (which is, of course, the Rybczynski cone in the present case). The maximum output producible at A is the height of the OCD surface $A\tilde{A}$, which is less than the marginal-variational valuation $A\bar{A}$ for 0A. The over-valuation of the residual vector thus causes an understatement of the opportunity cost of (the factors used in) the project.

Figure 5(b) extends this construction to incorporate two goods, x_1 and x_2. If we select units such that the output prices are unity for both

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[1] The directional cosines at v reflect the marginal products of the two inputs. Their ratio is the slope of line CD in plane $v_1 0 v_2$, while PP'//CD.

goods, the surfaces $0C\bar{q}_1$ and $0D\bar{q}_2$ reflect what is producible if all inputs are used to produce only x_1 and x_2, respectively, and their convex hull therefore has surface OCD and tangent plane OPP' reflecting the marginal-variational valuation of all residual factor vectors. The factor proportions producing good x_1 (x_2) under the prevailing factor prices before the project are reflected by ray $0q_1$ ($0q_2$). If a project leaves a residual factor vector 0A within the Rybczynski cone $q_1 0 q_2$, then the marginal-variational valuation $A\bar{A}$, thereof, agrees with the true maximum output value producible therewith: there is therefore no overvaluation for 0A and thus no understatement of the project cost.

Figure 5(c) modifies Figure 5(b) to allow international prices to differ from domestic prices, as in the FWSB distortion. Thus, it portrays an export tax on commodity x_1. Consequently, the world market value of any output mix is higher than the domestic market value if and only if some x_1 is produced. The $0C\hat{q}_1\bar{q}_2D$ surface is constructed to reflect this fact. The $0\hat{q}_1\bar{q}_2$ planar segment reflects therefore the true social value of any residual factor vector falling within the Rybczynski cone $q_1 0 q_2$.

We can now proceed to our full illustration of the 3-good, 2-factor, FWSB-distortion case. Thus Figures 5(d)-(e) additionally portray output of x_3 which is also assumed, like x_1, to be exported under an export tax. It is further assumed that the factor-intensity of good x_3 lies between that of the other two goods. Clearly the construction of the $0C\hat{q}_1\hat{q}_3\bar{q}_2D$ surface is similar to that of the $0C\hat{q}_1\bar{q}_2D$ surface in Figure 5(c).

Figure 5(d) then shows the case where, by coincidence, $\hat{q}_1\hat{q}_3\bar{q}_2$ fall on one line such that $0\hat{q}_1\hat{q}_3\bar{q}_2$ constitutes a single planar segment whose height at any residual factor vector within the Rybczynski cone $0q_1q_2$ then reflects the true social value of that vector. Hence also the difference between

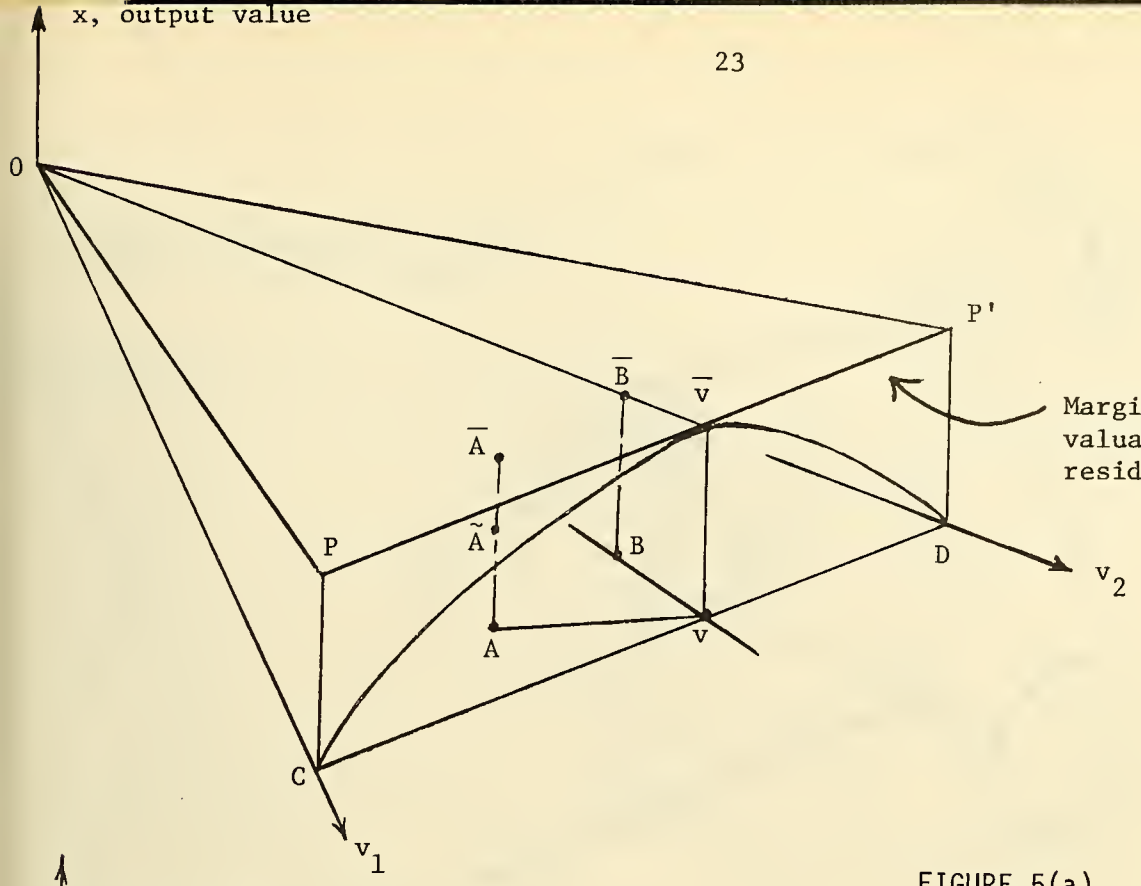


FIGURE 5(a)

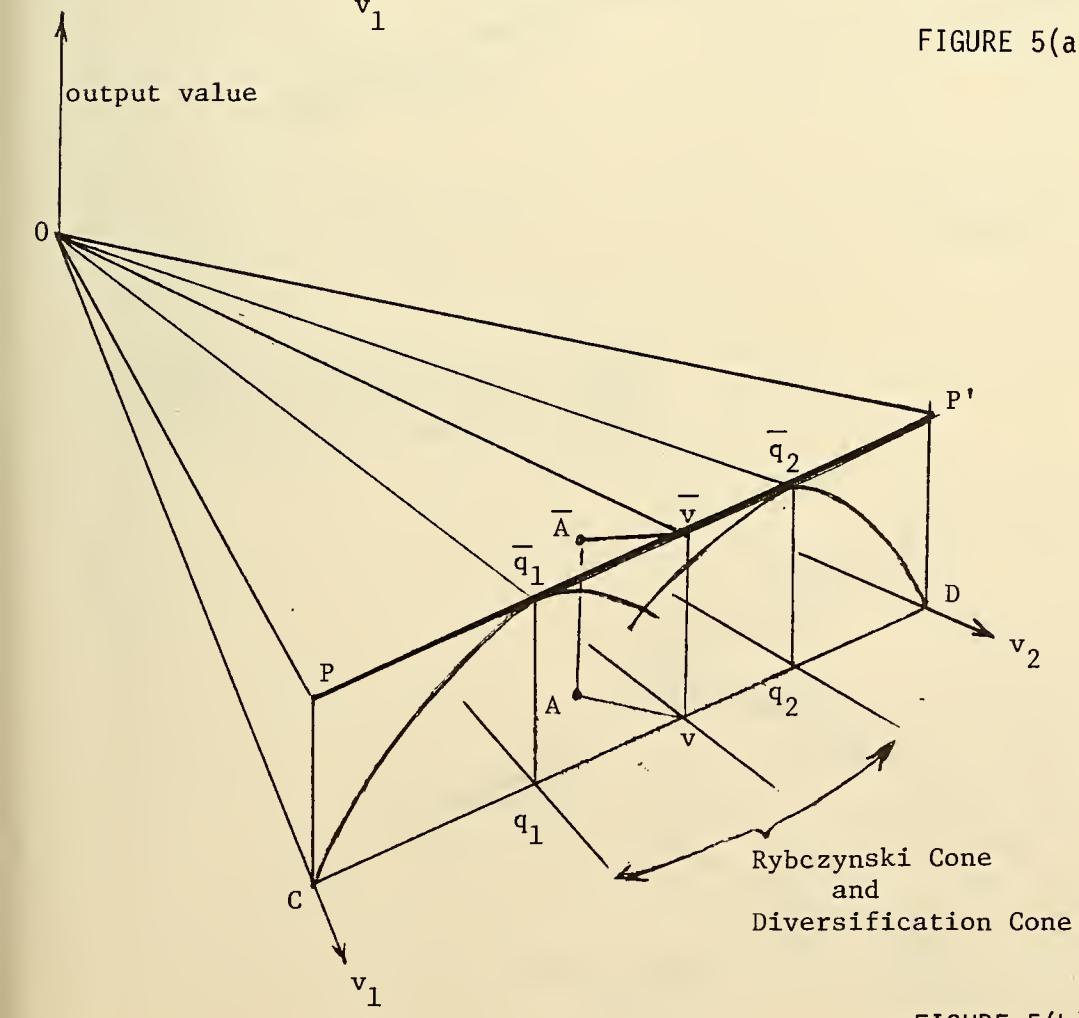
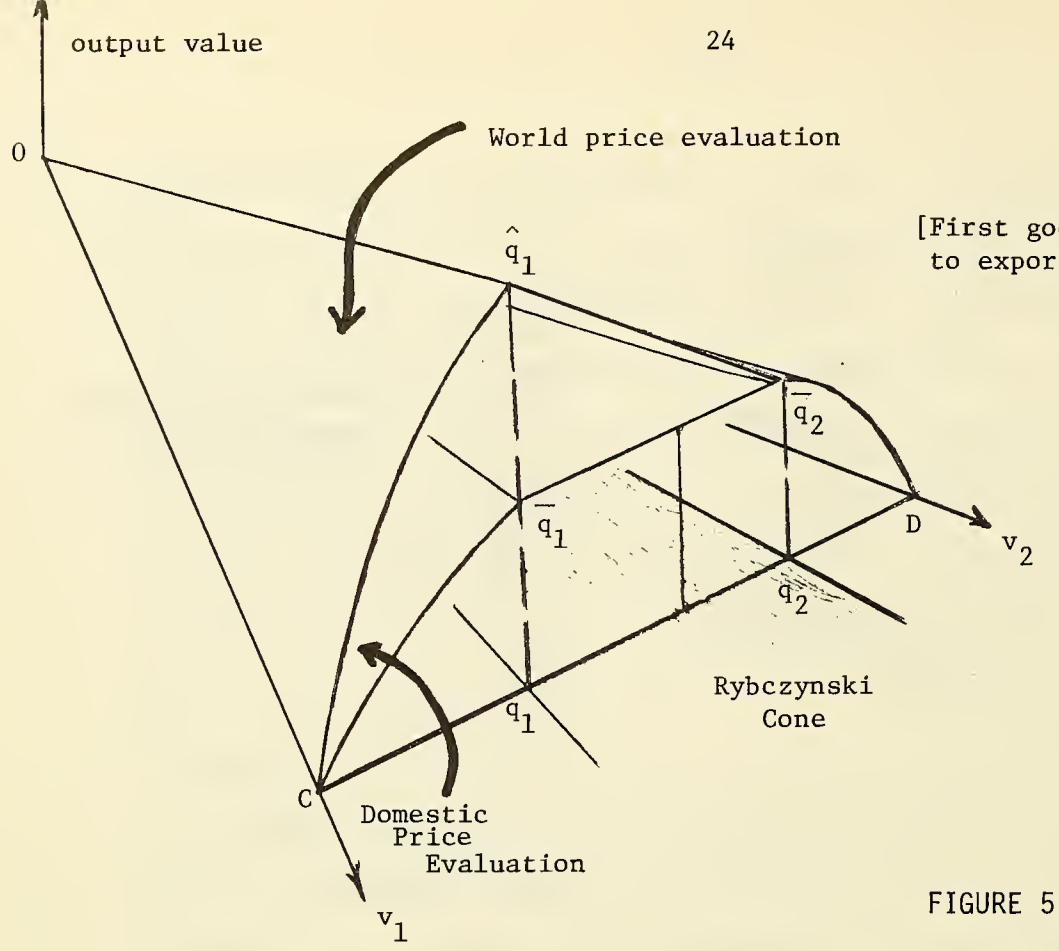
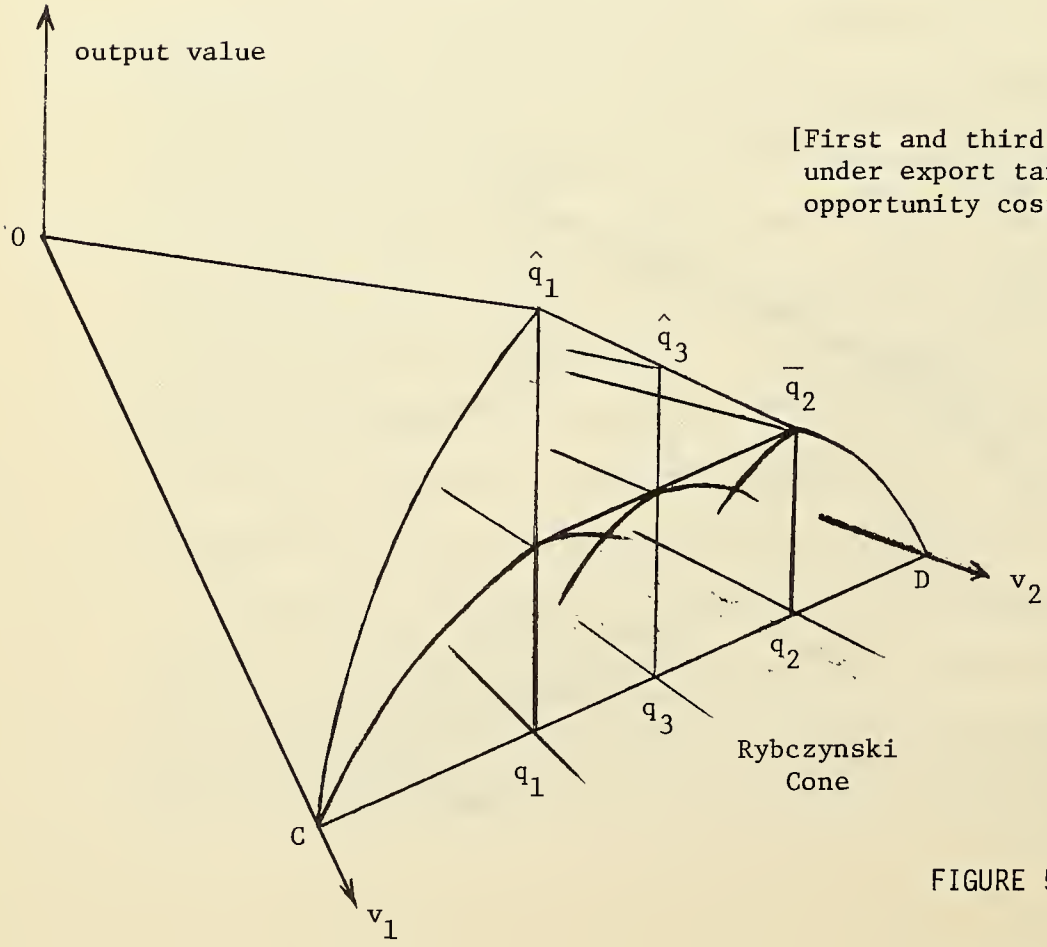


FIGURE 5(b)



[First good exported subject to export tax]

FIGURE 5(c)



[First and third goods exported under export taxes; by coincidence, opportunity cost is unique]

FIGURE 5(d)

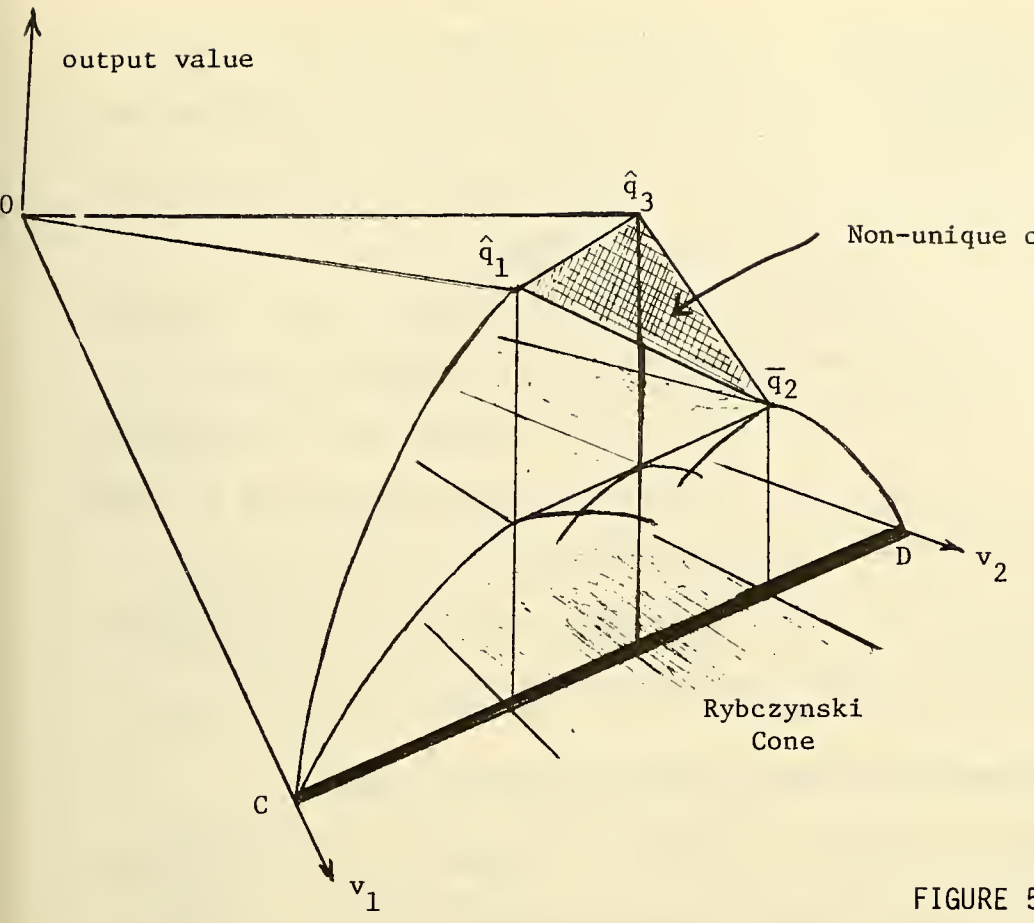

output value
0
$\hat{q}_3$
$\hat{q}_1$
Non-unique opportunity cost
$\bar{q}_2$
D
v_2
Rybczynski
Cone
C
v_1

FIGURE 5(e)

that height and the height $v\bar{v}$ (corresponding to the pre-project endowment vector) represents the true opportunity cost of the factors withdrawn for project use: and clearly, because of the planar segment, this opportunity cost is unique even within the Rybczynski cone.

By contrast, the general case is depicted in Figure 5(e). Here the planar segment disappears and instead one gets indeterminate, social opportunity cost for the project within the Rybczynski cone. Depending on the output mix, we can see that the (social) value producible by a residual factor vector can vary from the "optimistic surface" $0\hat{q}_1\hat{q}_3\bar{q}_2$ to the "pessimistic surface" $0\hat{q}_1\bar{q}_2$; and the project evaluator cannot predict what will happen and therefore which opportunity cost will prevail.[1]

(B) Case III: Factors Outnumber Goods plus No Distortion

We now turn to the case where factors outnumber goods and there is no distortion. As one would expect from the wellknown work of Samuelson (1953) and other writers in the theory of international trade, even this distortion-free world will present problems because, in general, primary factor prices will vary with the factor endowments so that the withdrawal of factors for project use will generally imply varying factor prices and hence the absence of a set of stationary shadow factor prices except for a negligible set of residual factor vectors. More precisely, we will argue the following propositions, illustrating them diagrammatically here and relegating the formal proofs to Appendix I, concerning the implications of the case where factors outnumber goods:

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[1] On the other hand, note that the uniqueness of opportunity cost will return once we leave the Rybczynski cone.

(a) Shadow factor prices will be stationary if and only if the residual factor proportions belong to that negligible set of all possible proportions[1] where the residual factors can be absorbed into industries at preproject factor intensities.

(b) If the condition in (a) is violated, the Law of Variable Proportions will cause the opportunity cost for the project to differ from the marginal variational value of the project,[2] and to exceed the latter by a Secondary Cost.

(c) This Secondary Cost will bear a proportional relation to the marginal variational value of the residual factors: a proportion rising steadily from zero at a nondecreasing rate as the residual factor proportions deviate progressively from the "negligible set" cited in (a).

(d) There is a continuum of diversification sub-cones, each of them polyhedral in form and corresponding to a unique domestic factor price vector. If that domestic factor price vector deviates from the pre-project factor price vector, then all residual factor vectors in that sub-cone will be overvalued under (pre-project) marginal-variational valuation by the same proportion. Each diversification sub-cone is a Rybczynski cone (defined on some domestic factor price vector) and the totality of all such diversification cones, not necessarily convex, forms the McKenzie-Chipman diversification cone.

Propositions (a)-(c) can be visualized immediately by reference to a 2-factor, 1-good model. In Figure 6(a), if the residual factor vector lies (as at A) on the ray OR from the origin, which is the ray on which the

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[1] This set is the lower-dimension cone of non-negatively weighted sums of the pre-project factor proportions for various industries.

[2] I.e. of the factors withdrawn.

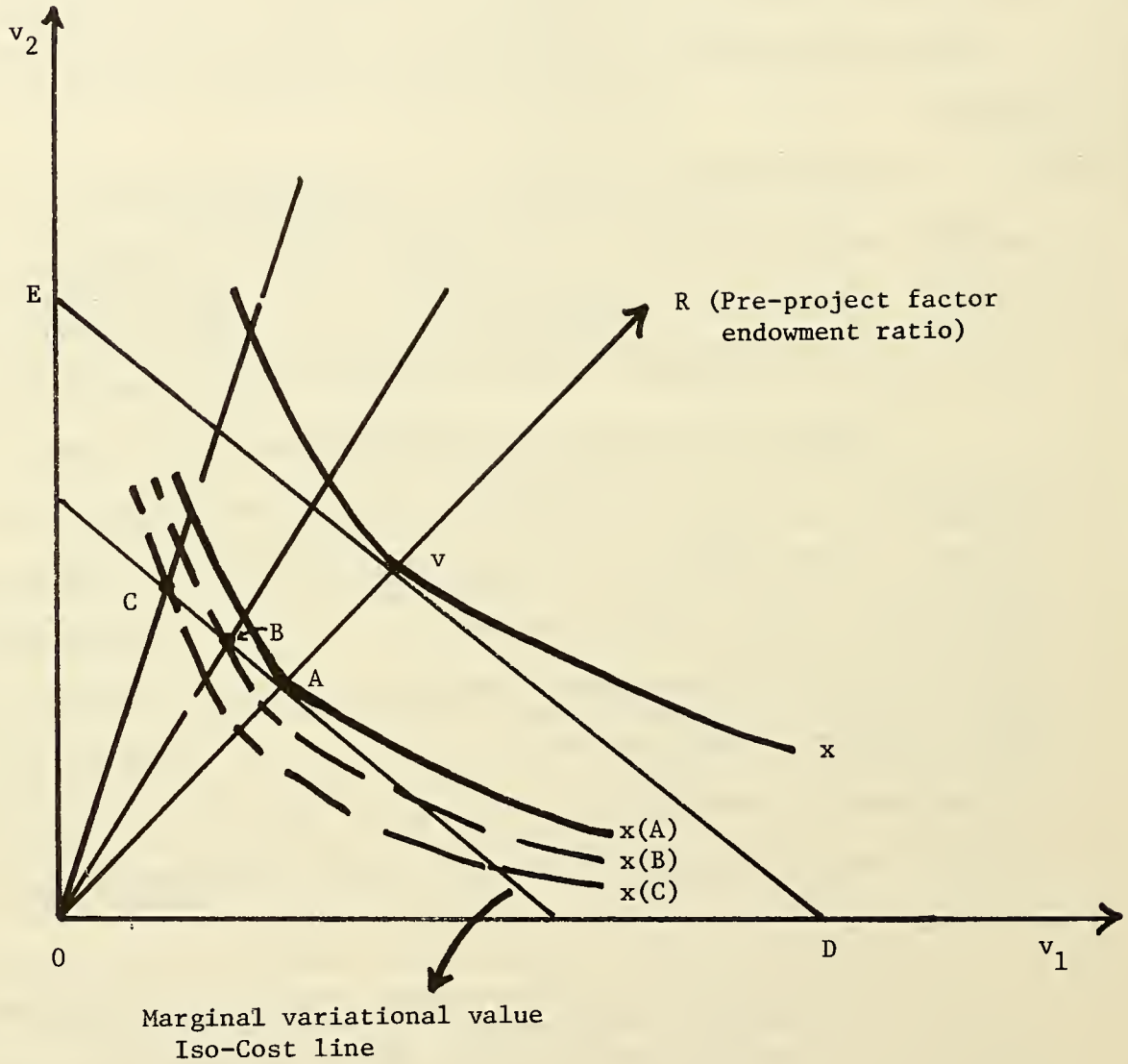
v_2
E
R (Pre-project factor endowment ratio)
v
C
B
A
x
x(A)
x(B)
x(C)
0
D
v_1
Marginal variational value
Iso-Cost line

FIGURE 6(a)

pre-project factor endowment ratio lies, clearly there will be no project-induced change in the factor prices. For projects that leave the residual factor endowment ratios elsewhere (as at B and C, for example, on the rays OB and OC), on the other hand, there will be corresponding changes in factor prices and the secondary cost that results may be illustrated in the now-familiar construction (see Figure 5(a)) in Figure 6(b). Note, in particular, that the values of the outputs for the isoquants passing through A, B, C and indeed all points along the ABC line in Figure 6(a) can be plotted as the concave curve in Figure 6(b) and this at once illustrates proposition (c) above.[1] At the same time, note that the residual factor proportions must lie on the ray OR if stationarity of shadow factor prices is to be maintained: thus clearly demonstrating the "negligible" character of such stationarity-preserving projects in our two-dimensional world, as argued in propositions (a) and (b) above.

Turn now to the 3-factor, 2-good world where proposition (d) is also admissible and hence demonstrable. Since the geometry here is somewhat less intuitive, and the formal proof in Appendix I is even less so except to mathematical economists, a few words of intuitive explanation are in order at the outset.

Thus, corresponding to the initial pre-project factor endowments, assume that the factor prices are determined such that all goods are produced (i.e. "diversification" obtains). Now, if for a finite project the post-project residual factor endowment vector can be accommodated by suitable reweighting of the initial factor proportions vectors, this is fine and

---

[1] Recall that the curve x(A) x(B) x(C) in Figure 6(b) is actually the vertical cross-section of the production surface along the line ABC in Figure 6(a): a fact which we will exploit in our 3-factor, 2-good illustration to follow.

stationarity obtains: though, as stated in proposition (a), this set of possibilities is "negligible." Suppose now that the residual factor endowment vector requires new factor prices and hence new factor proportions vectors in the production of the different goods. We can then immediately see both that there is now secondary cost and that, assuming continuing diversification, generally speaking there will be again a "negligible" set of factor-withdrawal possibilities at which the new factor prices and proportions, and hence secondary cost vis-a-vis the old (marginal-variational) factor prices, will remain unchanged. Therefore, clearly proposition (d) is intuitively established.

Turning now to our geometry in Figure 7, assume that we choose the unit of money such that the pre-project national output is valued at unity and we then choose units for each output and each factor such that their unit prices also equal unity. In Figure 7, the factor quantities are then measured along axes $0v_1$, $0v_2$ and $0v_3$ respectively. Point D (respectively, E and F) represents factor 1 (respectively, factors 2 and 3) of one unit, which is worth one unit of value, i.e. the same as the pre-project national output at pre-project factor prices. DEF is then the triangle for all factor bundles with unit-value at the pre-project factor prices. The initial factor endowment vector v then must clearly lie on this triangle since, by definition, its value equals that of national output. Let then the pre-project equilibrium factor proportions be such that, in good x_1, the factor vector that would lie on DEF is (.4,.5,.1) and, in good x_2, it is (.1,.5,.4) units of factors v_1, v_2 and v_3. Figure 7 illustrates the former as point q^1, depicting its coordinates with a cut-away on the triangle DEF, whereas the latter is illustrated as q^2. Evidently, the aggregate factor endowment v will lie on the linear segment q^1q^2. Equally, it is

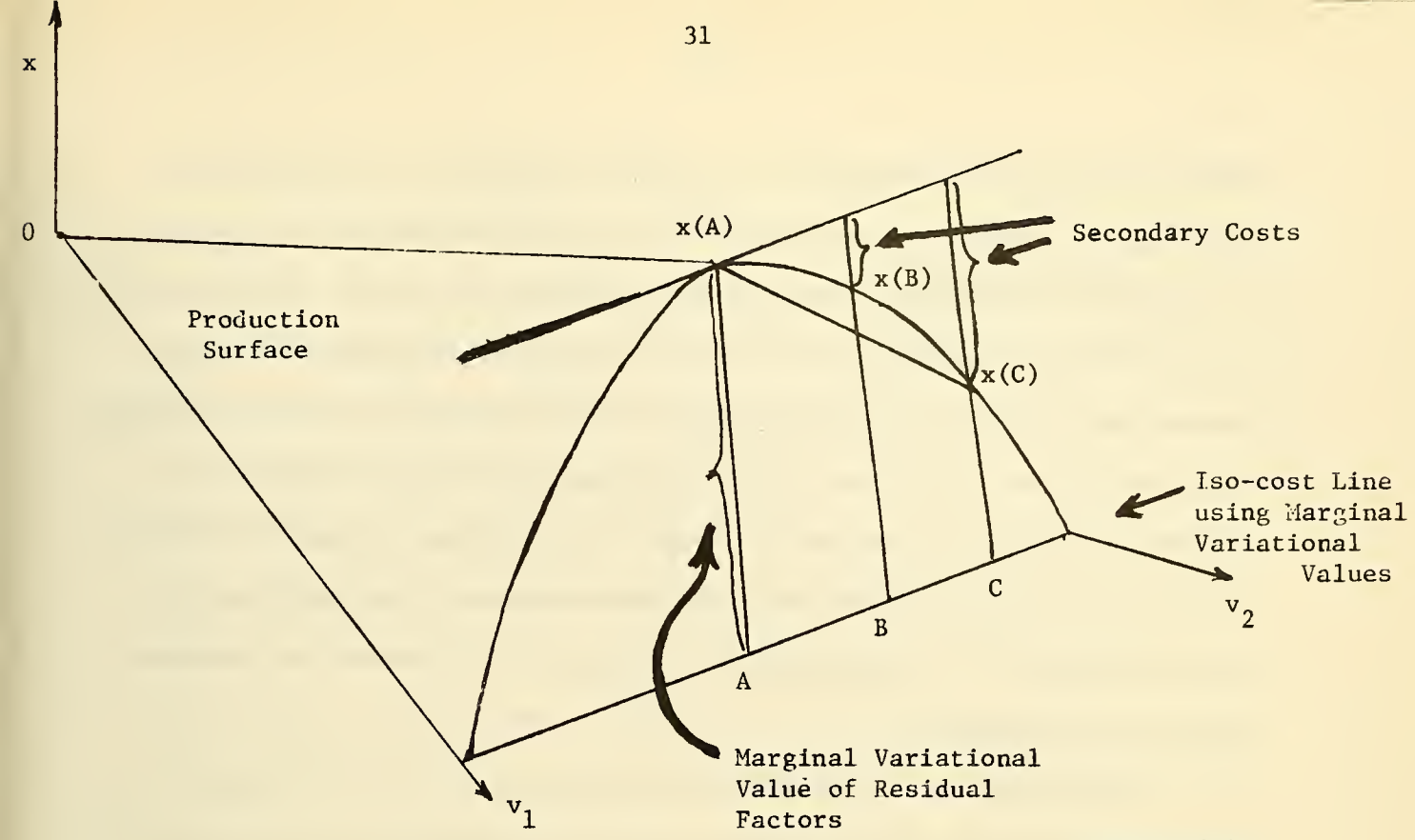

FIGURE 6(b)

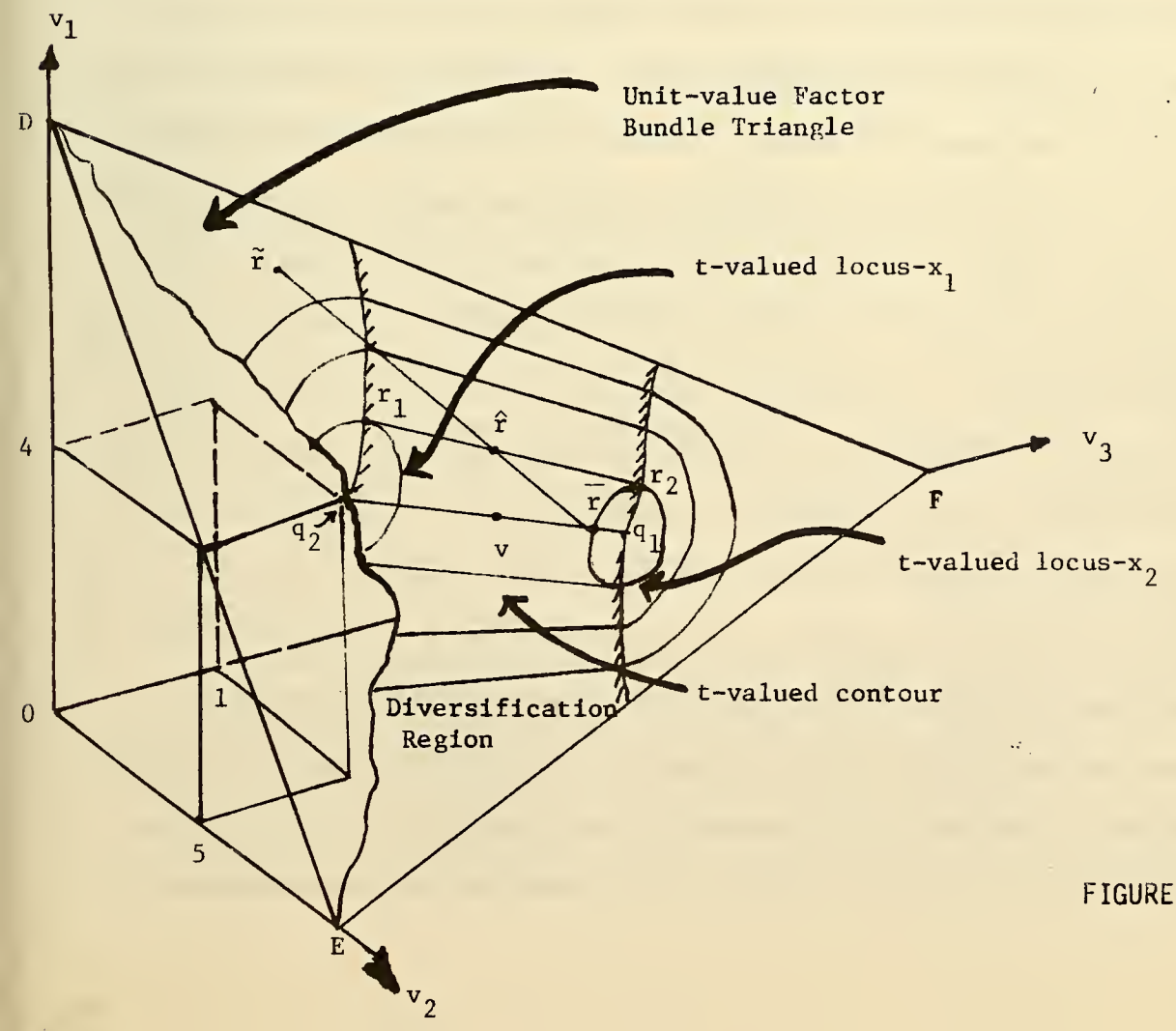

FIGURE 7

clear that the two-dimensional cone $q^1 0 q^2$ (not drawn) is the Rybczynski cone and, as long as the residual factor vector lies within it (i.e. such that the residual factor vector is a weighted sum of the $0q^1$ and $0q^2$ vectors), there will be no secondary cost and the pre-project marginal-variational factor prices will yield the true opportunity cost of the factors withdrawn for project use. [Note that, given constant returns, only proportions matter; hence <u>any</u> residual vector that is proportional to some vector $0\bar{r}$ where $\bar{r}$ is on the triangle DEF within the Rybczynski cone, will also be accurately priced by the marginal-variational factor prices and secondary cost will not arise.]

Suppose now, however, that at any point such as $\bar{r}$, on $q^1 q^2$, one produces <u>only</u> good x_2 (rather than the linear combination of goods x_1 and x_2 for which the marginal-variational valuation <u>would</u> give, as we have already seen, the true opportunity cost). In this case, the factor bundle $0\bar{r}$ will be overvalued if evaluated at the marginal-variational prices, by a proportion (1-t) say, as identically would some bundle, e.g. r_2, off the $q^1 V q^2$ linear segment. Joining then all the bundles in DEF where, only good x_2 being produced, the output value will amount to a fraction t of its marginal-variational valuation, we therefore obtain the t-valued locus - x_2 in Figure 7. Analogously, assuming that only good x_1 is produced, the t-valued locus - x_1 can be obtained. Assume now that r_1 and r_2 are the tangent points of these two loci with their common tangent. Then the marginal-variational valuation of input bundle $\hat{r}$ on $r_1 r_2$ will incur an over-valuation of (1-t); and any project leaving a residual vector $\hat{v}$ proportional to $\hat{r}$ will incur a secondary cost of (1-t) times the marginal-variational value of $\hat{v}$. The boundary of the convex-hull of the t-valued loci for x_1 and x_2 is then clearly a "t-valued" contour, three members of

such contours being shown in Figure 7. Quite generally, for any project leaving a residual vector v proportional to $0\tilde{r}$ will incur a secondary cost equal to: [the t-index at $\tilde{r}$] x [the marginal-variational valuation of the residual factor vector]. Moreover, noting in Figure 7 that $\bar{r}\hat{r}\tilde{r}$ fall on one line, observe that the secondary cost proportion (relative to the marginal-variational value of the residual factor vector) at $\tilde{r}$ is greater than at $\hat{r}$, which latter is greater than that at $\bar{r}$, namely, 0. Finally, note that the totality of all common tangents to the product-specific iso-t loci, like r_1r_2, constitutes the diversification region, which need not be convex. As long as a residual factor vector is proportional to any point in this region, both goods will be produced. But only when the residual factor vector is proportional to a point in q^1q^2, will secondary cost disappear. The fact that q^1q^2 is a proper subset to the diversification region marks the difference between the case (III) where factors outnumber goods and the cases (I-II) where goods are not outnumbered by factors.

(C) Case VI: Factors Outnumber Goods plus FWSB Distortion

Here, as in Case III, except for a negligible set of residual factor vectors, shadow factor prices may be non-stationary. However, unlike Case III, the presence of FWSB distortion makes it impossible to draw any conclusions about the secondary cost--its magnitude, or even its sign[1]--once the residual factor vector falls outside the Rybczynski cone.

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[1] In common with Cases IV and V, the marginal-variational valuation of a project may overstate as well as understate the project cost, once the residual factor vector is outside the Rybczynski cone. Thus, Case V, as illustrated in Figure 5(c), demonstrates this possibility. When a residual factor vector is within certain parts of the q_2OD cone, the extension of the $0\hat{q}_1\bar{q}_2$ plane may cut under the $0\bar{q}_2D$ surface, so that the shadow prices will undervalue the residual factor vector and a negative secondary cost will follow.

Section IV: Concluding Remarks

Our analysis leads to many observations.

First, as a quick perusal of the summary of Table 1 on the many goods and factors cases will show, the relative numbering of factors and goods is of significance. Project evaluation by shadow price computation is possible in the happy world of $m = n$ and between the Scylla of indeterminacy where $m > n$ and the Charibdis of nonstationarity where $m < n$. Of course, both factor and product classifications are only Arbeitsidée invoked for our convenience. Classifications are suitable only when intraclass variations (or own elasticities) are much smaller (or show stronger substitutabilities) than the interclass ones. In a general model with continuua of factors and products, both indeterminacy and nonstationarity may co-exist.

Second, our analysis has clear applicability to the transfer problem, conceived as, not a transfer of purchasing power but rather as, a transfer of factors of production as may be the case when reparations payments have to be made in barter (e.g., Soviet Union transferring factories from Germany after World War II). By contrast with the purchasing-power-shift variety of transfers, analyzed in the standard trade-theoretic literature (e.g. by Samuelson (1952) (1954), Johnson (1956) and others) we must now contend with the possible existence of secondary costs even when the small-country assumption is made. Furthermore, while in the former case there can be secondary cost or gain, in our case there can only be a secondary cost.

Third, our analysis also has applicability therefore to the theory of international factor mobility. Thus, the existing theoretical analyses, by Grubel-Scott (1966), Berry-Soligo (1969), Johnson (1968) and Kenen (1971), of the welfare effects of brain drain on "those left behind," as reviewed

and synthesized in Bhagwati and Rodriguez (1975), relate to two-factor models with one or two products, with focus mainly on one-factor emigration. Our present analysis leads, however, to the following generalization:

Result 4.1: For a small country without distortions, any finite level of emigration of factors, singly or in combination, will harm (or have no effect on) those left behind, if the residual factor vector falls outside (or is left inside) the pre-emigration Rybczynski cone. The harmful effect will almost always obtain when factors outnumber goods.

Note that the analysis can be readily extended to the cases with FWSB distortion, mutatis mutandis.[1]

Fourth, for a small country, without distortion, the welfare impact of finite factor increases (e.g., labor immigration, capital inflow) on those originally present is completely symmetrical. There is no effect or there is some harmful effect, depending upon whether the "augmented factor vector" is inside or outside the Rybczynski cone, and the latter possibility will almost always arise with factors outnumbering goods.

Fifth, the locus classicus of shadow prices is mathematical programming. It appears desirable to relate our findings with the programming framework, cross-referencing the economic assumptions, the programming formulation, the shape of the guyot profile and the implications for the shadow cost of a project. All these are tabulated in Table 2. Note that, inside the Rybczynski cone, the Law of Variable Proportions is held at bay for the small-country, constant-returns situation; hence stationarity of shadow

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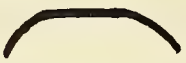
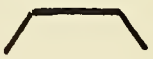
[1]The extension to the "large country" case can also be undertaken, as in Appendix II below.

Table 2: Relationship of Analysis to Mathematical Programming

Set of Economic Assumptions	Programming Framework	Guyot Profile	Secondary Cos
(1) Large country, and/or Decreasing returns to scale	Nonlinear programming	Flat-top absent	Ubiquitous
Small country (2) Constant returns to scale Variable coefficients	Generalized linear-programming of Dantzig-Wolfe (cf. Dantzig (1963))	Flat-top with smoothly declining descent 	Absent if residual vect stays inside the Rybczynsk cone
(3) Small country Fixed coefficients	Linear programming	Flat-top with linearly declining descent 	Absent if residual vect stays inside the Rybczynsk cone

*Analysis is based upon the distortion-free case. With FWSB distortion, results hold, mutatis mutandis.

cost obtains. Under fixed proportions (linear programming), the t-index decreases at a constant rate outside the Rybczynski cone. Under variable proportions, the t-index declines imperceptibly at first outside the Rybczynski cone, but then the decline accelerates at increasing speed.

Finally, lest the reader question the utility of our analysis of the no-distortions cases, partly because the world is largely characterized by distortions and partly because, when it is not, it may be contended that there is no ground for undertaking project analysis, let us remark that: first, there are indeed countries such as Hong Kong and Singapore where the no-distortions model may be a more reasonable approximation to reality; second, where secondary costs obtain, as when the entry of

foreign firms is on a massive scale (as in South Korea and Taiwan) and raises local wages, project evaluation would appear necessary (unless one believes, as we do not, that the market will generate and disseminate the "true" shadow prices automatically in such cases).

Appendix I: The Analytics of the Many Goods and Factors Cases

Is opportunity cost always uniquely defined for a project? Can shadow prices derived by marginal variation ever represent exactly the opportunity cost for large projects (i.e. structural change)? The answers are no and yes, in brief. Proofs and details follow.

I: The Distortion-Free Cases (I-III):[1] The Framework

Let $x, p \in R^m$ be the vectors of pre-project outputs and their prices, $v \in R^n$ be the pre-project vector of factor supply, and $f_j(\cdot)$ the no-joint output production function for good j, which is increasing, concave and linear homogeneous in the input vector $\tilde{v}^j$, $j = 1,\ldots,m$.

In a small, open economy where domestic prices equal world prices, the competitive output and input usage vectors $(x, (v^1,\ldots,v^m))$ maximize the Dantzig-Wolfe Generalized Linear Programme (GLP):

$$(1) \quad \begin{cases} \text{Max} \quad p \cdot \tilde{x} \\ \tilde{x} \geq 0,\ \tilde{v}^j \geq 0 \\ \text{subject to:} \quad \tilde{x}_j = f_j(\tilde{v}^j), \quad j = 1,\ldots,m \\ \qquad\qquad \sum_1^m v^j = v \end{cases}$$

Let S(v) be the set of all optimal, feasible output vectors and g(v) be the maximized value of the objective function for (1). The observed pre-project factor prices $w \in R^n$ represent the dual vector for (1), where $v^j/x_j = (a_{ij},\ldots,a_{nj})$, the unit input vector for good j, is cost-minimizing under w. Hence the input-output matrix will be written as:

$$A(w) = [a_{ij}(w)]$$

Let $\Delta_k v \leq 0$ be the factor vector withdrawn for project k, $(x+\Delta_k x)$ be a solution in $S(v+\Delta_k v)$, and $(w+\Delta_k w)$ be the associated factor price vector. The competitive duality is:

	Pre-project	Post-project	
(2)	$A(w)x = v$	$A(w+\Delta_k w)(x+\Delta_k x) = v + \Delta_k v$	(Resource Allocation)
(3)	$w'A(w) = p'$	$(w+\Delta_k w)'A(w+\Delta_k w) = p'$	(Income Distribution)

p is unchanged by the project since the economy is "small." $g(v+\Delta_k v)$ is always unique, although $(x+\Delta_k x)$ may not be. The opportunity cost of the project k is:

$$c_k = g(v) - g(v+\Delta_k v) = -p \cdot \Delta_k x.$$

II: The FWSB Distortion Cases (IV-VI): The Framework

Now, let $p^* \in R^m$ be the world output price vector: $p^* \neq p$. The competitive duality remains the same as in (2)-(3). National output however will be evaluated at p^*. For each v, the output value at p^* ranges over:

$$G(\tilde{v}) = [\underline{G}(\tilde{v}), \overline{G}(\tilde{v})] = [\min_{S(\tilde{v})} p^* \cdot \tilde{x}, \max_{S(\tilde{v})} p^* \cdot \tilde{x}], \text{ say.}$$

The opportunity cost of project k ranges over $[\underline{c}_k, \overline{c}_k] = [p^* \cdot x - \overline{G}(v+\Delta_k v), p^* \cdot x - \underline{G}(v+\Delta_k v)]$. Uniqueness obtains if $G(v+\Delta_k v)$ contains a single value, $g^*(v+\Delta_k v)$, and $c_k = g^*(v) - g^*(v+\Delta_k v) = -p^* \cdot \Delta_k x$.

III: Additional Concepts

To avoid non-essential complexities, three regularity conditions are imposed:

(4) $x_j > 0$, all j, $w_i > 0$, all i (vectorial condition)

(5) $A(\tilde{w})$ is single valued, containing only positive entries and with full rank: min (m,n) at all $\tilde{w}$ (matric condition)

(6) $F_j(\cdot)$ is continuously differentiable, $j = 1,\ldots,m$ (functional condition)

Ordinary linear programmes do not assume (6), hence the gradient ∇g is undefined there at "degeneracies."

By (5)-(6), we have:

(7) $$w = \nabla g(v) \qquad w + \Delta_k w = \nabla g(v + \Delta_k v).$$

When $g^*(\tilde{v})$ is defined everywhere, we have analogously:

(8)[2] $$w^* = \nabla g^*(v) \qquad w^* + \Delta_k w^* = \nabla g^*(v + \Delta_k v)$$

For convenience, we now define the Rybczynski Cone as:

$V(w) = \{\tilde{v} \geq 0$: there exists $\tilde{x} \geq 0$, $A(w)\tilde{x} = \tilde{v}\}$, a convex cone of dimension min (m,n) in the non-negative orthant of R^n (w will be omitted below).

Then, a project is a V-project [N-project, respectively] if it leaves a residual vector $(v + \Delta_k v)$ inside V [resp., outside V]. A $\mathring{V}$-project is a V-project with associated $(v + \Delta_k v) = A(w)\tilde{x}$, where $\tilde{x}_j > 0$, all j.

By (7)-(8), we further have:

(9) $$c_k = \begin{cases} \int_{v+\Delta_k v}^{v} \tilde{w} \cdot d\tilde{v}, & \tilde{w} = \nabla g(\tilde{v}) \quad \text{for cases I-III} \\ \int_{v+\Delta_k v}^{v} \tilde{w}^* \cdot d\tilde{v}, & \tilde{w}^* = \nabla g^*(\tilde{v}) \quad \text{for cases IV-VI if } g^*(\tilde{v}) \text{ exists.} \end{cases}$$

IV: Problems and Observations

Problem of uniqueness--when is c_k uniquely defined?

Problem of stationarity / representability--when is $\tilde{w}$ (resp. $\tilde{w}^*$) constant on some path joining v to $(v+\Delta_k v)$ so that (9) reduces to:

$$(10) \qquad c_k = \begin{cases} -w \cdot \Delta_k v & \text{for Cases I-III} \\ -w^* \cdot \Delta_k v & \text{for Cases IV-VI if } g^*(\tilde{v}) \text{ exists} \end{cases}$$

and what can be deduced of the bias: $c_k + w \cdot \Delta_k v \neq 0$ for Cases I-III?

V: Observations on Uniqueness

(i) (Opportunity cost is unique, if there is no distortion or factors outnumber goods.)

For Cases I-III, V-VI, c_k is unique.

(ii) (Opportunity cost is not unique over a non-negligible set, unless by coincidence, if goods outnumber factors under distortion.)

For $\mathring{V}$-projects in Case IV, unless $p - p^*$ is orthogonal to the subspace containing $S(v+\Delta_k v)$, c_k is non-unique. Given p in the simplex of normalized price vectors (a set of dimension $m-1$), p^* must be in a linear set of dimension $n-1$ for c_k to be unique.

Proof:

Part (i): For Cases I-III, $g(\cdot)$ is always defined uniquely. Hence $c_k = g(v) - g(v+\Delta_k v)$ is uniquely defined. For Cases V-VI, by (5), $A(\tilde{w})$ is of rank $m \leq n$, for any $\tilde{w}$. If $S(v+\Delta_k v)$ contains two distinct members $x^{(1)}$ and $x^{(2)}$, then by (2), $A(\tilde{w})(x^{(1)} - x^{(2)}) = 0$ for some $\tilde{w}$. Set A_0 to be any non-singular $m \times m$ sub-matrix of $A(\tilde{w})$, then, $0 = A_0^{-1}(0) = x^{(1)} - x^{(2)} \neq 0$: a contradiction. Therefore, $S(v+\Delta_k v)$ has only one member. $g^*(v+\Delta_k v)$ exists

and c_k is unique.

Part (ii): For project k which is a $\mathring{V}$-project, $(v+\Delta_k v)\varepsilon V$ and $A(w)\tilde{x} = (v+\Delta_k v)$ with $\tilde{x}_j > 0$, all j. Since A(w) has m columns, $n < m$ of which are linearly independent, $S(v+\Delta_k v)$ is of dimension $(m-n)$. If c_k is to be unique, then for any $x^{(1)}, x^{(2)} \varepsilon S(v+\Delta_k v)$, we should have $p^* (x^{(1)} - x^{(2)}) = 0 = p (x^{(1)} - x^{(2)})$, i.e. $(p-p^*)$ perpendicular to $S(v+\Delta_k v)$. In the normalized price simplex of dimension $(m-1)$, this means that for given p, p* must happen to be on an $(n-1)$ dimensional linear set.

Remark: If there is no unique c_k, it is possible that some project (or no project!) may have either positive or negative opportunity cost: w* cannot be defined usefully.

VI: Observations on Stationarity/Representability

(i) (Stationarity and representability prevail for V-projects, unless goods outnumber factors under distortion.)

For V-project in Cases I-III (resp. V-VI), $\tilde{w}$ (resp. $\tilde{w}^*$) is identical to w (resp. w*) along the linear segment joining v to $v+\Delta_k v$, hence $c_k = -w \cdot \Delta_k v$ (resp. $c_k = -w^* \cdot \Delta_k v$).

(ii) (If there is no distortion, the opportunity cost is underestimated (resp. over-estimated) by the value of factors withdrawn at pre-project (resp. post-project) marginal variational factor prices, for all N-projects[3].)

For N-projects in Cases I-III, $-w \cdot \Delta_k v < c_k < -(w+\Delta_k w) \cdot \Delta_k v$; also $(c_{k'} + w \cdot \Delta_{k'} v) \geq h(c_k + w \cdot \Delta_k v)$ for $\Delta_{k'} v = h \Delta_k v$ and $h > 1$.

(iii) (Among the N-projects under no distortion, secondary cost increases at least proportionally, when factor withdrawn differs directionally more from a V-project. For one project whose factor withdrawal is the mean of two others, secondary cost for the mean project is no more than the mean

secondary cost.)

For projects 1, 2 and 3, if $\Delta_2 v = h\Delta_1 v + (1-h)\Delta_3 v$, $h\in[0,1]$, $c_2 + w\Delta_2 v \leq h(c_1 + w\Delta_1 v) + (1-h)(c_3 + w\Delta_3 v)$.

(iv) (V-projects are negligible when factors outnumber goods, with or without distortion.)

V being of dimension min (m,n), in Cases III, VI, a V-project must happen to leave a residual vector in the m-dimensional V out of the n-dimensional non-negative orthant.

Proof:

Part (i): Consider the family of Generalized Linear Programmes: $\{GLP(h): 0 \leq h \leq 1\}$ where $v + h\Delta_k v$ replaces v in (1), for GLP(h). It is easily verified that the optimal, feasible primal-dual pair for GLP(h) is: $\left((x + h\Delta_k x,\ A(w)\ \text{diag.}\ (x_1 + h\Delta_k x_1, \ldots, x_m + h\Delta_k x_m))\right)$. Hence, $g(v + h\Delta_k v) = p^{\bullet}(x + h\Delta_k x)$ [resp. $g^*(v + h\Delta_k v) = p^{*\bullet}(x + h\Delta_k x)$, when $g^*(\bullet)$ is defined] and $\nabla g(v + h\Delta_k v) \equiv w$ [resp. $\nabla g^*(v + h\Delta_k v) \equiv w^*$ when $g^*(\bullet)$ is defined]. Therefore, (10) follows (9) for any V-project, say, project k.

Part (ii): First we prove a Lemma:

Lemma: $g(\bullet)$ is concave:

(11) $\quad g(v^{(1)}) + (v^{(2)} - v^{(1)}) \cdot \nabla g(v^{(1)}) - g(v^{(2)}) \geq 0$ for any $v^{(1)}$, $v^{(2)}$.

Proof of Lemma: Since $g(\bullet)$ is linear homogeneous, by Euler's Theorem, (11) becomes:

(12)
$$\begin{aligned} 0 &\leq v^{(2)} \cdot \nabla g(v^{(1)}) - g(v^{(2)}) && \text{(By (7), } g = v\cdot\nabla g\text{)} \\ &= w^{(1)} v^{(2)} - p\cdot x^{(2)} && \text{(By (7) and def. of g)} \\ &= w^{(1)}\cdot A(w^{(2)})x^{(2)} - w^{(1)}\cdot A(w^{(1)})x^{(2)} && \text{(By (2) and (3))} \\ &= w^{(1)'}[A(w^{(2)}) - A(w^{(1)})]x^{(2)} \end{aligned}$$

where $w^{(1)}$ and $w^{(2)}$ are the factor price vectors associated with $v^{(1)}$ and $v^{(2)}$ respectively, $x^{(2)}$ is an output vector associated with $v^{(2)}$. (12) always holds, $A(w^{(1)})$ being cost-minimizing, hence the row vector $w^{(1)'}[A(w^{(2)}) - A(w^{(1)})]$ is non negative; but then the column vector $x^{(2)}$ is also non-negative, so that their inner product must be non-negative as well. Therefore, (11) is proved. Reverting now to the proof of Part (ii), and setting $v = v^{(1)}$ and $v + \Delta_k v = v^{(2)}$ in (11), we have $c_k \geq -w \cdot \Delta_k v$. But by (6), $A(w^{(2)}) \neq A(w^{(1)})$, differing column for column when $w^{(2)} \neq w^{(1)}$. So $w^{(1)'}[A(w^{(2)}) - A(w^{(1)})]$ has all components positive. But $x^{(2)}$ has some positive component and no negative component. Hence, (12) is a strict inequality. Consequently, $c_K > -w \cdot \Delta_k v$. Setting $v = v^{(2)}$ and $v + \Delta_k v = v^{(1)}$, similarly, one proves $c_k < -(w + \Delta_k w) \cdot \Delta_k v$.

Part (iii): For (iii), on the other hand, $c_k + w\Delta_k v = [g(v) + (-\Delta_k v)\nabla g(v)] - g(v + \Delta_k v)$ is a convex function of $(v + \Delta_k v)$, since it is the difference between an affine function and a concave function as the concavity of g follows the Lemma.

Part (iv): V is the image of the m-dimensional non-negative orthant under the mapping of A(w) into the n-dimensional non-negative orthant and $m < n$. Unless $(v + \Delta_k v)$ happens to be in this low-dimension cone, the project is not a V-project.

Remark: It is easily shown that, with distortion, c_k and w_i^* for some i can be negative. Also $g^*(\cdot)$ is no longer concave. [Cf. Bhagwati-Srinivasan-Wan (1977)]

VII: Generalizations and Extensions

The above model, suitably modified, can accommodate (i) joint outputs, (ii) traded inputs and (iii) non-traded, domestically produced inputs unfit for consumption. In case that (iv) there are primary goods with variable supply, or (v) some non-traded domestic goods are consumable, our results may remain applicable even for projects with finite scale. The precise limits of applicability of our results, however, cannot be determined with the presence of (iv) or (v) without solving an explicit general equilibrium problem. This is however no different from the question what is the limit of validity for our small-country assumption. A brief sketch of these issues follows:

Extensions (i-iii). Assume there are m' industries each with a constant returns, concave production function F^j involving a vector for tradeables (i.e. both traded products and traded inputs), $\tilde{x}^j$, a vector of domestically produced non-traded inputs, $\tilde{y}^j$, and the vector of non-traded primary inputs, $\tilde{v}^j$, for industry $j = 1,\ldots,m'$. Constraint (1) is then replaced with:

$$(1') \qquad \begin{cases} F^j(\tilde{x}^j,\tilde{y}^j,\tilde{v}^j) = 0 \\ \sum_{j=1}^{m'} (\tilde{x}^j,\tilde{y}^j,\tilde{v}^j) = (\tilde{x},0,v) \end{cases}$$

where $\tilde{x}$ is the net export vector of tradeables for the economy. The Hicks-Debreu convention is followed in (1') to assign inputs (outputs) with negative (positive) signs in the summation. Results in V-VI above have been dependent upon the number of primary factors (n) versus the number of outputs (m). We now only have to replace the number of outputs (m) by the number of industries (m'), and all earlier results hold. Thus the dimension

of the Rybczynski cone V is min (m',n) rather than min (m,n) and hence non-stationarity becomes nearly ubiquitous if $m' < n$, while in a distortion-ridden economy, non-uniqueness threatens if $m' > n$. The numbers of outputs, per se, whether tradeable or non-tradeable but unfit for consumption are irrelevant for the stationarity and uniqueness results. So is the number of traded inputs .

Extensions (iv-v). To allow for domestically consumed, non-traded good and variably supplied primary inputs, we have to replace the second equality in (1') with

$$(1'') \qquad \sum_{1}^{m'} (\tilde{x}^j, \tilde{y}^j, \tilde{v}^j) = (\tilde{x}, \tilde{y}, \tilde{v})$$

where $\tilde{y}$ is the vector of consumption for non-tradeables and $\tilde{v}$ is the vector of non-traded, primary factor supply by the household sector. Our results on uniqueness depends upon whether $\tilde{v}$, $\tilde{v} + \Delta_k \tilde{v}$ belong to the interior of V. [If the answer is yes: indeterminacy.] Our results on stationarity and representability depends upon whether the vector $(\tilde{v} + \Delta_k \tilde{v})$ remains in V. [If the answer is yes: stationarity.] There is no reasonable way--short of solving a general equilibrium problem--to decide the precise magnitudes of $\Delta_k \tilde{v}$. $\Delta_k \tilde{v}$ can be decomposed into two terms: direct project-use $\Delta_k v$ and induced household variation $(\Delta_k \tilde{v} - \Delta_k v)$. It is the latter term defying simply prediction. Yet it is definitely untrue that any finite variation $\Delta_k \tilde{v}$ (caused by any small but finite project) will thrust the residual factor vector across the boundary of V. Casual empiricism may be enough in many practical situations.

The complexities of (iv-v) are due to the market response of the household sector, while all earlier analyses are conducted in terms of the

technology of the business sector. If the household sector is "small" relative to the business sector in that its response does not affect the crossing and noncrossing of certain threshhold (is $\tilde{v}+\Delta_k\tilde{v}$ in $\mathring{V}$ or outside V?), the complexity makes no difference.

Footnotes to Appendix I:

[1]For easy reference, the six cases referred to below are the following:

Cases	No Distortion	With (FWSB) Distortion
Goods outnumbering factors	I	IV
Goods equal factors	II	V
Goods outnumbered by factors	III	VI

"Distortion" means that world prices differ from domestic prices due to tariffs, subsidy, etc.

[2]Unlike w, w* cannot be observed on the market. When $n < m$, $g^*(\cdot)$ is not generally defined and w* cannot be usefully defined either. For $n = m$, w* may also be derived from $w^{*\prime} = p^{*\prime}A(w)^{-1}$: an approach inapplicable for $n > m$. Here, v also enters directly in determining w*.

[3]One might define tautologically, shadow prices for finite change:

$$\hat{w}_i(\Delta_k v) = \begin{cases} \int_{v}^{v+\Delta_k v} \tilde{w}_i \, d\tilde{v} & \text{for } \Delta_k v_i < 0 \\ w_i \text{ or } (w_i + \Delta_k w_i) \text{ [or any finite number!]} & \text{for } \Delta_k v_i = 0 \end{cases}$$

so that $c_k \equiv \Sigma_i \hat{w}_i(\Delta_k v_i)$, for Cases I-III. Similarly for Cases V-VI, $\hat{w}_i^*(\Delta_k v)$ can be defined analogously. But such contrivances would be useless to predict any opporunity cost, ex ante.

Appendix II: Welfare Implications of Opportunity Cost, Compensated Trading Gains, Consumption Distortions and the Large-Country Case

A few well-known facts in welfare economics and trade are summarized here, by reference to three questions:

(1) Throughout our analysis, the undistorted opportunity cost, $-p^*\Delta x$, is used as a welfare measure; in what sense is this measure meaningful?

(2) Findlay-Wellisz (1976), Srinivasan-Bhagwati (1976), Clemhout-Wan (1970) and others have not analyzed the case of consumption distortion; to what extent can we do that?

(3) What can be said if the country is not small in Samuelson's sense?

Fact 1: In an economy facing fixed external prices and no consumption distortions, an increase in aggregate income signifies the new situation promises potential Paretian gain in the sense that someone can be made better off and no one need be worse off under a system of appropriate welfare transfers as Samuelson (1962), Vanek (1965), Grandmont-McFadden (1972) and Kemp-Wan (1972) have demonstrated. It is always possible to make all persons except one just as well off as before, but one person better off. Or in the tradition of Bhagwati and Johnson (1960) as well as Wan (1965), one can made each person as well as before and still leave over a "maximum bonus". No global existence of social indifference curves is needed in such welfare comparisons.

Fact 2: If consumption distortion is present, compensated gain is possible, given an increase of aggregate income at the fixed world prices. After all, one can postulate the existence of (N+1) persons in an

N-person small country, where persons 1 to N are "natural persons" having preferences as in reality and the 0th person has parallel hyper-planes as indifference curves with directional cosines proportional to either the world prices, or distorted domestic prices or whatever set of "semi-positive" (i.e., all nonnegative but some positive) prices. The 0th person initially (i.e., before project initiation) has zero income. If we keep the first N persons (0th through (N-1)st) at their initial respective consumption menus, any rise of aggregate income at world prices can be transferred to the Nth person (the Princeps a la Kemp-Wan) to be allocated along his Engel curve, corresponding to the distorted price, to achieve the highest satisfaction for him. This position can never be worse off for the Princeps relative to his status quo, ex ante position --- one can never be hurt by an incremental dosage of income, even if that incremental income can only be spent on various goods at distorted prices. This shows the fact that compensated gain under distortion will be possible when aggregate, undistorted national income increases. But if we hold the last N-persons (1st to Nth) at their menus at status quo, ex ante, the 0th person's gain (i.e., the Bhagwati-Johnson measure of Maximum Bonus) will be possible when aggregate undistorted national income increases. World prices (and hence distorted prices) being constant, holding a person's income at the initial level by lump-sum transfers will hold his menu constant.

Fact 3: The only difference that a distortion on consumption can make is that if some goods are inferior, at distorted prices, then starting from some initial, distorted equilibrium, all consumers may still gain even when aggregate, undistorted national income decreases ---

a point independently noted by Vanek (1965), Bertrand (1969) and Bhagwati (1968). To convince the reader, one may simply visualize in a 2-good world the distorted Engel's curve for an individual as an S-shaped curve with the distorted budget line and the undistorted budget line arranged as in Figure 8 below. Income reduction valued at world prices may push this individual from position C' to C", which increases his welfare. If distributional gain benefits all other individuals, one has then witnessed the paradoxical case where a reduction in aggregate undistorted income is accompanied by universal welfare gain for everyone under competitive conditions.

Fact 4: So far we have confined ourselves to the small country case; what can be said for a large country? Obviously, whatever difference that can arise (and by assumption, must arise) comes through the terms of trade effect. Stationarity of output prices is lost; only by sheer coincidence can the shadow factor prices stay stationary.

The next question is: what kind of bias would the use of marginal-variational output prices bring? Such a case can be tackled within the 2-country, 2-good context, but we shall directly go to the general case, of an arbitrary number of countries and goods.

We note first that, by a theorem of Krueger and Sonnenschein (1967), a price change is beneficial if the before-project trade vector yields an export surplus at after-project prices, i.e., if

$$z\Delta p > 0,$$

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[1] In fact, in the terminology of Bhagwati (1971), the large country assumption with laissez faire is a "foreign distortion", so that we are now indeed considering a case with distortion.

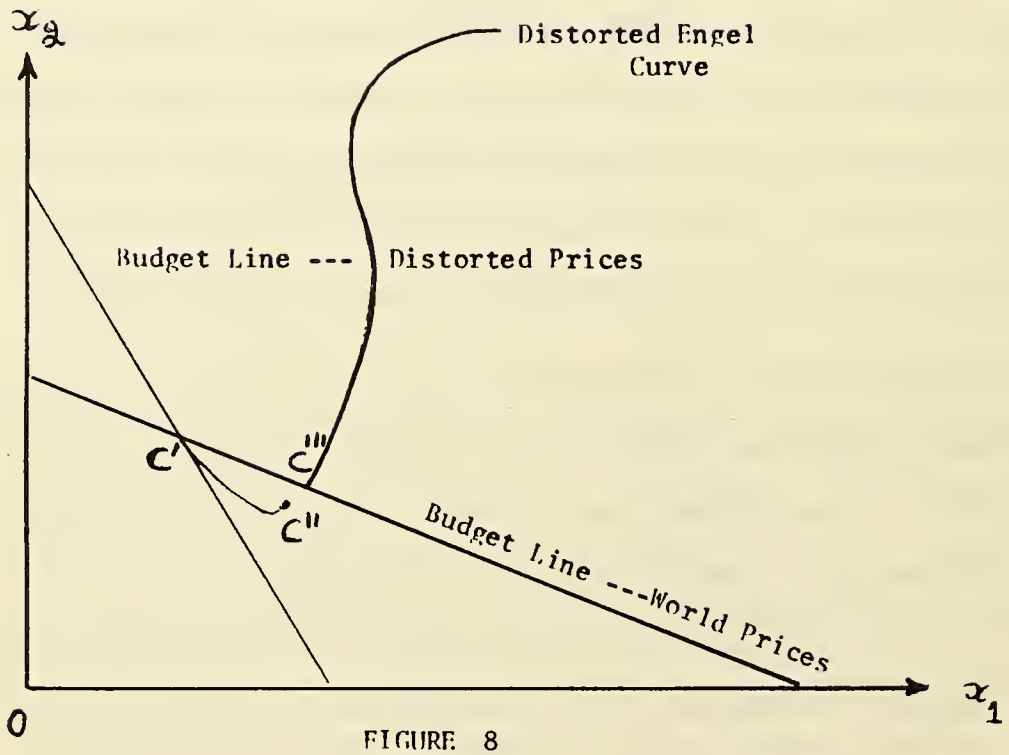
x_2
Distorted Engel Curve
Budget Line --- Distorted Prices
C'''
C'
C''
Budget Line ---World Prices
x_1
0

FIGURE 8

where z is the before-project trade vector (i.e., the net export vector). This provides us with a sufficient criterion, that a small enough project has a true opportunity cost lower than the marginal-variational opportunity cost $p\Delta x$, if the following holds:

$$z \cdot \pi(\Delta x) > 0$$

where $\pi(\Delta x)$ is the price variation Δp which is induced by Δx. Note that the Krueger-Sonnenschein criterion is a sufficient but not a necessary condition.

Figure 9 below summarizes a "general case" where the "small-country" assumption is valid for a certain nonnegligible, proper subclass of all possible projects, identified by their respective input requirements. In the 2×2 world, the quadrilateral set OABC represents all ordered nonnegative input pairs, the withdrawal of which would leave a nonnegative residual vector inside the Rybczynski cone. However, the Rybczynski cone is constructed by assuming factor proportions for the two goods to remain such as to minimize the production costs at marginal-variational input prices under the assumption that output prices remain fixed. Once the small country assumption becomes inapplicable, nonstationary output prices cause nonstationary input prices via the Stolper-Samuelson theorem. Consequently, only within the set Oaa'bb'cc', the intersection between OABC and the small-country region, will the stationarity of input prices hold.

Once output prices vary, two possibilities may materialize. (a) The terms of trade may improve; this is the Krueger-Sonnenschein zone where the opportunity cost is partially moderated via the terms of trade effect. (b) The terms of trade may deteriorate; here three cases must be distinguished: (i) the "direction of trade" remains the same; (ii) autarky occurs; and

(iii) the trade pattern is reversed. Within case (iii), there is even a subcase, where the terms of trade change favors the country in question (the "anti-Krueger-Sonnenschein zone") even though the terms of trade deteriorate in the sense of Laspeyres. The country can now export its erstwhile importable good at such advantage as to make the price-effect favorable again. [In the very short run, this last outcome is remote. Of course, in longer run, matters can be different: Prussia and Russia were grain exporters until the recent centuries.] The situation becomes quite different for a world with more than 2 goods. There is much scope for the terms of trade effect to be favorable while the Laspeyres index deteriorates: variation in the composition of exports is by no means uncommon. [Figure 2 of Kemp and Wan (1972) depicts such a case quite clearly.]

The irregular shapes of the "small country region" (shaded), the autarky locus, etc., reflect a key fact --- as the Sonnenschein-Debreu findings indicate, almost any form of excess demand function is possible. Hence, without additional assumptions (or facts), little can be said.

But there is even one additional point to remember. Once output prices start to vary, then Δp may differ from project to project, even if they divert identical input vectors. Benefits cannot then be evaluated separately from cost and vice versa.

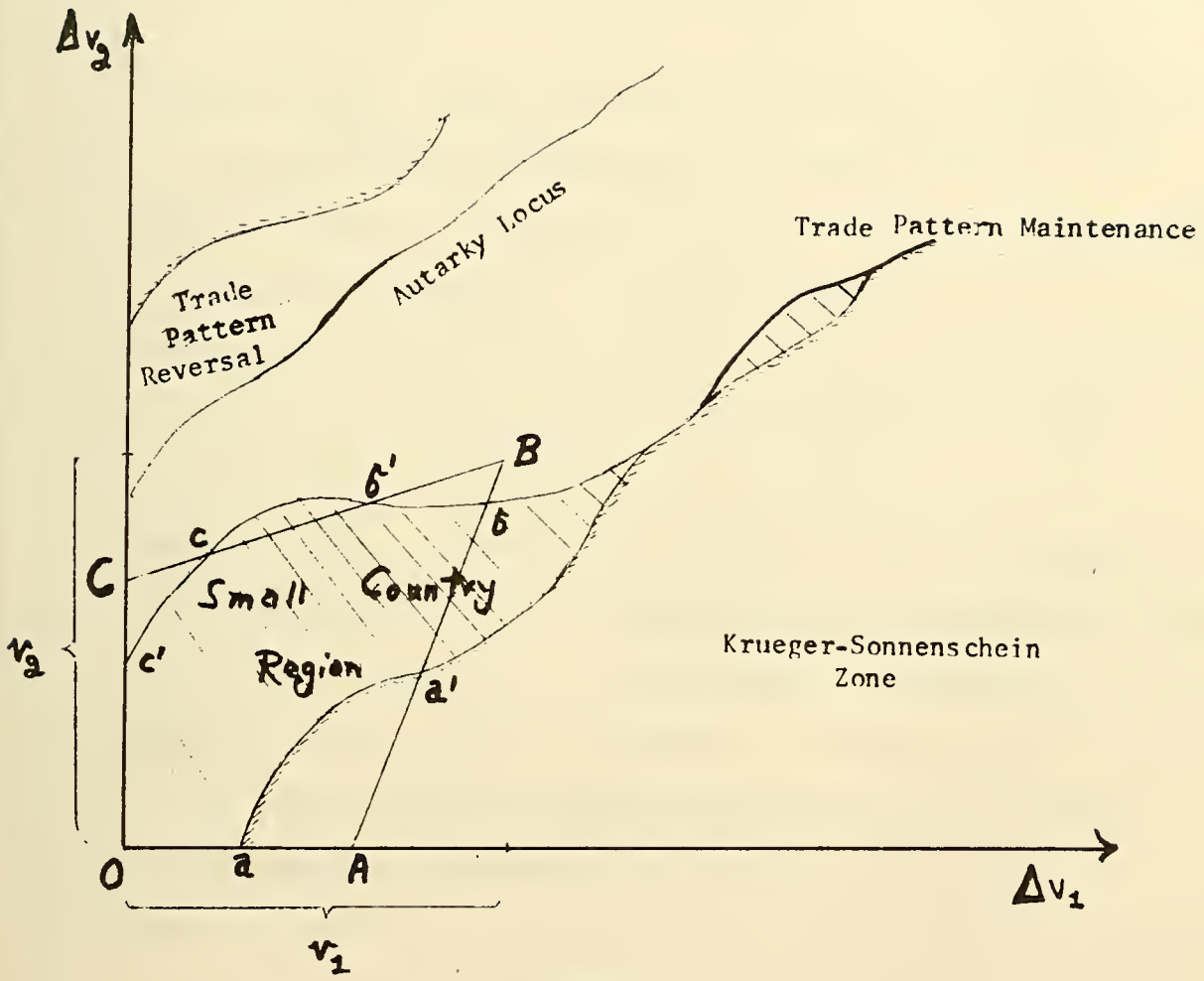
Δv_2
Trade Pattern Reversal
Autarky Locus
Trade Pattern Maintenance
B
b'
b
c
C
Small Country
Region
c'
v_2
a'
Krueger-Sonnenschein Zone
O
a
A
Δv_1
v_1

FIGURE 9

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